Progressions, Related Inequalities and Series

Choose the most appropriate option (a, b, c or d).

Q 1. If $a_1, a_2, a_3, \ldots\ldots$ are in AP then a_p, a_q, a_r are in AP if p, q, r are in

(a) AP (b) GP (c) HP (d) none of these

Q 2. Let t_r denote the rth term of an AP. If $t_m = \frac{1}{n}$ and $t_n = \frac{1}{m}$ then t_{mn} equals

(a) $\frac{1}{mn}$ (b) $\frac{1}{m} + \frac{1}{n}$ (c) 1 (d) 0

Q 3. If p, q, r, s $\in$ N and they are four consecutive terms of an AP then the pth, qth, rth, sth terms of a GP are in

(a) AP (b) GP (c) HP (d) none of these

Q 4. If in a progression $a_1, a_2, a_3, \ldots$, etc., $(a_r - a_{r+1})$ bears a constant ratio with $a_r \cdot a_{r+1}$ then the terms of the progression are in

(a) AP (b) GP (c) HP (d) none of these

Q 5. If $\frac{a_2 a_3}{a_1 a_4} = \frac{a_2 + a_3}{a_1 + a_4} = 3\left(\frac{a_2 - a_3}{a_1 - a_4}\right)$ then a_1, a_2, a_3, a_4 are in

(a) AP (b) GP (c) HP (d) none of these

Q 6. Let x, y, z be three positive prime numbers. The progression in which $\sqrt{x}, \sqrt{y}, \sqrt{z}$ can be three terms (not necessarily consecutive) is

(a) AP (b) GP (c) HP (d) none of these

Q 7. Let f(x) = 2x + 1. Then the number of real values of x for which the three unequal number f(x), f(2x), f(4x) are in GP I s

(a) 1 (b) 2 (c) 0 (d) none of these

Q 8. If $a_r > 0$, r $\in$ N and $a_1, a_2, a_3, \ldots\ldots., a_{2n}$ are in AP then

$$\frac{a_1 + a_{2n}}{\sqrt{a_1} + \sqrt{a_2}} + \frac{a_2 + a_{2n-1}}{\sqrt{a_2} - \sqrt{a_3}} + \frac{a_3 + a_{2n-2}}{\sqrt{a_3} + \sqrt{a_4}} + \ldots.. + \frac{a_n + a_{n+1}}{\sqrt{a_n} + \sqrt{a_{n+1}}}$$

is equal to

(a) $n-1$ (b) $\frac{n(a_1+a_{2n})}{\sqrt{a_1}+\sqrt{a_{n+1}}}$ (c) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_{n+1}}}$ (d) none of these

Q 9. If $a_1, a_2, a_3, \ldots\ldots, a_{2n+1}$ are in AP then

$$\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+\ldots.+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$$

is equal to

(a) $\frac{n(n+1)}{2}.\frac{a_2-a_1}{a_{n+1}}$ (b) $\frac{n(n+1)}{2}$ (c) $(n+1)(a_2-a_1)$ (d) none of these

Q 10. Let $a_1, a_2, a_3, \ldots.$ be in AP and a_p, a_q, a_r be in GP. Then $a_q : a_p$ is equal to

(a) $\frac{r-p}{q-p}$ (b) $\frac{q-p}{r-q}$ (c) $\frac{r-q}{q-p}$ (d) none of these

Q 11. If a, b, c are in GP then a + b, 2b, b + c are in

(a) AP (b) GP (c) HP (d) none of these

Q 12. If a,b,c,d are nonzero real numbers such that

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \leq (ab + bc + cd)^2$$

Then a, b, c, d are in

(a) AP (b) GP (c) HP (d) none of these

Q 13. If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are nonzero numbers, then a, b, c are in

(a) AP (b) GP (c) HP (d) none of these

Q 14. If a, b, c are in AP then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ are in

(a) AP (b) GP (c) HP (d) none of these

Q 15. If in an AP, $t_1 = \log_{10} a$, $t_{n+1} = \log_{10} b$ and $t_{2n+1} = \log_{10} c$ then a, b, c are in

(a) AP (b) GP (c) HP (d) none of these

Q 16. If n!, 3 × n! and (n + 1)! are in GP then n!, 5 × n! and (n + 1)! are in

(a) AP (b) GP (c) HP (d) none of these

Q 17. In an AP, the pth term is q and the (p + q)th term is 0. Then the qth term is

(a) $-p$ (b) p (c) $p + q$ (d) $p - q$

Q 18. In a sequence of $(4n + 1)$ terms the first $(2n + 1)$ terms are in AP whose common difference is 2, and the last $(2n + 1)$ terms are in GP whose common ratio is 0.5. If the middle terms of the AP and GP are equal then the middle term of the sequence is

(a) $\frac{n.2^{n+1}}{2^n - 1}$ (b) $\frac{n.2^{n+1}}{2^{2n} - 1}$ (c) $n \cdot 2^n$ (d) none of these

Q 19. If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$ then x, y, z are in

(a) AP (b) GP (c) HP (d) none of these

Q 20. If a, b, c, d and p are distinct real numbers such that

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$$

then a, b, c, d are in

(a) AP (b) GP (c) HP (d) none of these

Q 21. The largest term common to the sequences 1, 11, 21, 31, …. to 100 terms and 31, 36, 41, 46, …. to 100 terms is

(a) 381 (b) 471 (c) 281 (d) none of these

Q 22. The interior angles of a convex polygon are in AP, the common difference being 5°. If the smallest angle angles is $2\pi/3$ then the number of sides is

(a) 9 (b) 16 (c) 7 (d) none of these

Q 23. The minimum number of terms of 1 + 3 + 5 + 7 + ……. that add up to a number exceeding 1357 is

(a) 15 (b) 37 (c) 35 (d) 17

Q 24. In the value of 100! the number of zeros at the end is

(a) 11 (b) 22 (c) 23 (d) 24

Q 25. The sum of all the proper divisors of 9900 is

(a) 33851 (b) 23952 (c) 23951 (d) none of these

Q 26. The sum of all odd proper divisors of 360 is

(a) 77 (b) 78 (c) 81 (d) none of these

Q 27. In the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ..., where n consecutive terms have the value n, the 150^{th} term is

(a) 17 (b) 16 (c) 18 (d) none of these

Q 28. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ..., where n consecutive terms have the value n, the 1025^{th} term I s

(a) 2^9 (b) 2^{10} (c) 2^{11} (d) 2^8

Q 29. Let $\{t_n\}$ be a sequence of integers in GP in which $t_4 : t_6 = 1 : 4$ and $t_2 + t_5 = 216$. Then $t_1$is

(a) 12 (b) 14 (c) 16 (d) none of these

Q 30. If $\log\left(\frac{5c}{a}\right), \log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in AP, where a, b, c are in GP, then a, b, c are the lengths of sides of

(a) an isosceles triangle (b) an equilateral triangle

(c) a scalene triangle (d) none of these

Q 31. Let S be the sum, P be the product and R be the sum of the reciprocals of n terms of a GP. Then $P^2R^n : S^n$ is equal to

(a) 1 : 1 (b) (common ratio)^n : 1

(c) (first term)^2 : (common ratio)^n (d) none of these

Q 32. If the pth, qth and rth terms of an AP are in GP then the common ratio of the GP is

(a) $\frac{p+q}{r+q}$ (b) $\frac{r-q}{q-p}$ (c) $\frac{p-r}{p-q}$ (d) none of these

Q 33. The number of terms common between the series 1 + 2 + 4 + 8 + ... to 100 terms and 1 + 4 + 7 + 10 + ... to 100 terms is

(a) 6 (b) 4 (c) 5 (d) none of these

Q 34. The 10^{th} common term between the series 3 + 7 + 11 + ... and 1 + 6 + 11 + ... is

(a) 191 (b) 193 (c) 211 (d) none of these

Q 35. Three consecutive terms of a progression are 30, 24, 20. The next term of the progression is

(a) 18 (b) $17\frac{1}{7}$ (c) 16 (d) none of these

Q 36. If three numbers are in GP then the numbers obtained by adding the middle number to each of the three numbers are in

(a) AP (b) GP (c) HP (d) none of these

Q 37. If a_1, a_2, a_3 are in AP, a_2, a_3, a_4 are in GP and a_3, a_4, a_5 are in HP then a_1, a_3, a_5 are in

(a) AP (b) GP (c) HP (d) none of these

Q 38. If a, b, c, d are four numbers such that the first three are in AP while the last three are in HP then

(a) bc = ad (b) ac = bd (c) ab = cd (d) none of these

Q 39. If the first two terms of an HP be 2/5 and 12/23 then the largest positive term of the progression is the

(a) 6^{th} term (b) 7^{th} term (c) 5^{th} term (d) 8^{th} term

Q 40. If x, 2y, 3z are in AP, where the distinct numbers x, y, z are in GP, then the common ratio of the GP is

(a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

Q 41. If x > 1, y > 1, z > 1 are three numbers in GP then

$$\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$$

are in

(a) AP (b) HP (c) GP (d) none of these

Q 42. If a, a_1, a_2, a_3,. ..... a_{2n-1}, b are in AP, a, b_1, b_2, b_3,….,b_{2n-1}, b are in GP and a, c_1, c_2, c_3,….., c_{2n-1}, b are in HP, where a, b are positive, then the equation $a_n x^2 - b_n x + c_n = 0$ has its roots

(a) real and unequal (b) real and equal (c) imaginary (d) none of these

Q 43. If a, x, b are in AP, a, y, b are in GP and a, z, b are in HP such that x = 9z and a > 0, b > 0 then

(a) $|y| = 3z$ (b) $x = 3|y|$ (c) $2y = x + z$ (d) none of these

Q 44. If three numbers are in HP then the number obtained by subtracting half of the middle number from each of them are in

(a) AP (b) GP (c) HP (d) none of these

Q 45. a, b, c, d, e are five numbers in which the first three are in AP and the last three are in HP. If the three numbers in the middle are in GP then the numbers in the odd places are in

(a) AP (b) GP (c) HP (d) none of these

Q 46. Let $a_1, a_2, a_3, \ldots\ldots, a_{10}$ be in AP and $h_1, h_2, h_3, \ldots\ldots, h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then a_4h_7 is

(a) 2 (b) 3 (c) 5 (d) 6

Q 47. If in an AP, $S_n = p.\, n^2$ and $S_m = p.m^2$, where S_r denotes the sum of r terms of the AP, then S_p is equal to

(a) $\frac{1}{2}p^3$ (b) mnp (c) p^3 (d) $(m + n)p^2$

Q 48. If S_r denotes the sum of the $\frac{S_{3r} - S_{r-1}}{S_{2r} - S_{2r-1}}$ is equal to

(a) $2r - 1$ (b) $2r + 1$ (c) $4r + 1$ (d) $2r + 3$

Q 49. S_r denotes the sum of the first r terms of a GP. Then S_n, $S_{2n} - S_{3n} - S_{2n}$ are in

(a) AP (b) GP (c) HP (d) none of these

Q 50. If $(1 - p)(1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5) = 1 - p^6$, $p \neq 1$ then the value of $\frac{p}{x}$ is

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{1}{2}$ (d) 2

Q 51. If the sum of series $1 + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3} + \ldots\ldots$ to ∞ is a finite number then

(a) $x < 2$ (b) $x > \frac{1}{2}$ (c) $x > -2$ (d) $x < -2$ or $x > 2$

Q 52. Let S_n denote the sum of the first n terms of an AP. If $S_{2n} = 3S_n$ then $S_{3n} : S_n$ is equal to

(a) 4 (b) 6 (c) 8 (d) 10

Q 53. In a GP of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the GP is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$ (c) 4 (d) none of these

Q 54. In an AP, $S_p = q$, $S_q = p$ and S_r denote the sum of the first r terms. Then S_{p+q} is equal to

(a) 0 (b) –(p + q) (c) p + q (d) pq

Q 55. The coefficient of x^{15} in the product

$$(1 - x)(1 - 2x)(1 - 2^2 . x)(1 - 2^3 . x)......(1 - 2^{15} . x)$$

is equal to

(a) $2^{105} - 2^{121}$ (b) $2^{121} - 2^{105}$ (c) $2^{120} - 2^{104}$ (d) none of these

Q 56. The coefficient of x^{49} in the product $(x - 1)(x - 3) \dots (x - 99)$ is

(a) -99^2 (b) 1 (c) -2 500 (d) none of these

Q 57. If a, b, c are in AP then $a + \frac{1}{bc}, b + \frac{1}{ca}, c + \frac{1}{ab}$ are in

(a) AP (b) GP (c) HP (d) none of these

Q 58. The AM of two given positive numbers is 2. If the larger number is increased by 1, the GM of the numbers becomes equal to the AM of the given numbers. Then the HM of the given numbers is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) none of these

Q 59. Let a, b are two positive numbers, where a > b and 4 × GM = 5 × HM for the numbers. Then a is

(a) 4b (b) $\frac{1}{4}b$ (c) 2b (d) b

Q 60. If a, $a_1, a_2, a_3, \dots a_{2n}$, b are in AP and a, $g_1, g_2, g_3, \dots g_{2n}$, b are in GP and h is the HM of a and b then

$$\frac{a_1 + a_{2n}}{g_1 g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \dots + \frac{a_n + a_{n+1}}{g_n g_{n+1}}$$

is equal to

(a) $\frac{2n}{h}$ (b) 2nh (c) nh (d) $\frac{n}{h}$

Q 61. Let $a_1 = 0$ and $a_1, a_2, a_3, \dots, a_n$ be real numbers such that $|a_i| = |a_{i-1} + 1|$ for all i then the AM of the numbers $a_1, a_2, a_3, \dots a_n$ has the value A where

(a) $A < -\frac{1}{2}$ (b) $A < -1$ (c) $A \geq -\frac{1}{2}$ (d) $A = -\frac{1}{2}$

Q 62. Let there be a GP whose first term is a and the common ratio is r. If A and H are the arithmetic mean and the harmonic mean respectively for the first n terms of the GP, A . H is equal to

(a) a^2r^{n-1} (b) ar^n (c) a^2r^n (d) none of these

Q 63. If the first and the (2n – 1)th terms of an AP, a GP and an HP are equal and their nth terms are a, b and c respectively then

(a) a = b = c (b) $a \geq b \geq c$ (c) a + c = b (d) $ac - b^2 = 0$

Q 64. $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the HM between a and b if n is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

Q 65. If the harmonic mean between P and Q be H then $H\left(\frac{1}{P} + \frac{1}{Q}\right)$ is equal to

(a) 2 (b) $\frac{PQ}{P+Q}$ (c) $\frac{P+Q}{PQ}$ (d) $\frac{1}{2}$

Q 66. Let x be the AM and y, z be two GMs between two positive numbers. Then $\frac{y^3 + z^3}{xyz}$ is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) none of these

Q 67. If HM : GM = 4 : 5 for two positive numbers then the ratio of the numbers is

(a) 4 : 1 (b) 3 : 2 (c) 3 : 4 (d) 2 : 3

Q 68. In a GP of alternately positive and negative terms, any term is the AM of the next two terms. Then the common ratio is

(a) -1 (b) -3 (c) -2 (d) $-\frac{1}{2}$

Q 69. If a, b, c are in AP, and p, p' are the AM and GM respectively between a and b, while q, q' are the AM and GM respectively between b and c, then

(a) $p^2 + q^2 = p'^2 + q'^2$ (b) pq = p'q' (c) $p^2 - q^2 = p'^2 - q'^2$ (d) none of these

Q 70. If $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ then the minimum value of $\cos^3\theta + \sec^3\theta$ is

(a) 1 (b) 2 (c) 0 (d) none of these

Q 71. If a > 1, b > 1 then the minimum value of $\log_b a + \log_a b$ I s

(a) 0 (b) 1 (c) 2 (d) none of these

Q 72. The minimum value of $4^x + 4^{1-x}$, $x \in R$, is

(a) 2 (b) 4 (c) 1 (d) none of these

Q 73. If $x = \log_5 3 + \log_7 5 + \log_9 7$ then

(a) $x \geq \frac{3}{2}$ (b) $x \geq \frac{1}{\sqrt[3]{2}}$ (c) $x \geq \frac{3}{\sqrt[3]{2}}$ (d) none of these

Q 74. If $a_n > 1$ for all $n \in N$ then

$$\log_{a_2} a_1 + \log_{a_3} a_2 + \ldots\ldots\ldots + \log_{a_n} a_{n-1} + \log_{a_1} a_n$$

has the minimum value

(a) 1 (b) 2 (c) 0 (d) none of these

Q 75. The product of n positive numbers is 1. Their sum is

(a) a positive integer (b) divisible by n

(c) equal to $n + \frac{1}{n}$ (d) greater than or equal to n

Q 76. If x, y, z are three real numbers of the same sign then the value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ lies in the interval

(a) $[2, +\infty)$ (b) $[3, +\infty)$ (c) $(3, +\infty)$ (d) $(-\infty, 3)$

Q 77. The least value of $2\log_{100} a - \log_a 0.0001$, $a > 1$ is

(a) 2 (b) 3 (c) 4 (d) none of these

Q 78. If $0 < x < \pi/2$ then the minimum value of $(\sin x + \cos x + \operatorname{cosec} 2x)^3$ is

(a) 27 (b) 13.5 (c) 6.75 (d) none of these

Q 79. If x, y, z are positive then the minimum value of

$$x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y} \text{ is}$$

(a) 3 (b) 1 (c) 9 (d) 16

Q 80. a, b, c are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$. Then

(a) $a = b = \frac{1}{2}, c = \frac{1}{4}$ (b) $a = b = \frac{1}{4}, c = \frac{1}{2}$ (c) $a = b = c = \frac{1}{3}$ (d) none of these

Q 81. If a > 0, b > 0, c > 0 and the minimum value of

$$a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)$$

is λabc then λ is

(a) 2 (b) 1 (c) 6 (d) 3

Q 82. The value of $\sum_{n=1}^{10}\int_0^n x\,dx$ is

(a) an even integer (b) an odd integer (c) a rational number (d) an irrational number

Q 83. The sum of 0.2 + 0.004 + 0.00006 + 0.0000008 + ... to ∞ is

(a) $\frac{200}{891}$ (b) $\frac{2000}{9801}$ (c) $\frac{1000}{9801}$ (d) none of these

Q 84. If $(2n + r)r$, $n \in N$, $r \in N$ is expressed as the sum of k consecutive odd natural numbers then k is equal to

(a) r (b) n (c) r+ 1 (d) n + 1

Q 85. $\sum_{r=1}^{n} r^2 - \sum_{m=1}^{n}\sum_{r=1}^{m} r$ is equal to

(a) 0 (b) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$ (c) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r\right)$ (d) none of these

Q 86. If $(1 + x)(1 + x^2)(1 + x^4)....(1 + x^{128}) = \sum_{r=0}^{n} x^r$ then n is

(a) 255 (b) 127 (c) 63 (d) none of these

Q 87. The value of $\sum_{n=1}^{m}\log\frac{a^{2n-1}}{b^{m-1}}$ $(a \neq 0, 1;\ b \neq 0, 1)$ is

(a) $m\log\frac{a^{2m}}{b^{m-1}}$ (b) $\log\frac{a^{2m}}{b^{m-1}}$ (c) $\frac{m}{2}\log\frac{a^{2m}}{b^{2m-2}}$ (d) $\frac{m}{2}\log\frac{a^{2m}}{b^{m+1}}$

Q 88. The sum of the products of the ten numbers $\pm1, \pm2, \pm3, \pm4, \pm5$ taking two at a time is

(a) 165 (b) -55 (c) 55 (d) none of these

Q 89. The sum of the series $\frac{1}{\log_2 4}+\frac{1}{\log_4 4}+\frac{1}{\log_8 4}+......+\frac{1}{\log_{2^n} 4}$ is

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)(2n+1)}{12}$ (c) $\frac{1}{n(n+1)}$ (d) none of these

Q 90. If $\sum_{n=1}^{n} n, \frac{\sqrt{10}}{3}.\sum_{n=1}^{n} n^2, \sum_{n=1}^{n} n^3$ are in GP then the value of n is

(a) 2 (b) 3 (c) 4 (d) nonexistent

Q 91. The value of $\sum_{r=1}^{n}\left\{(2r-1)a+\frac{1}{b^r}\right\}$ is equal to

(a) $an^2+\frac{b^{n-1}-1}{b^{n-1}(b-1)}$ (b) $an^2+\frac{b^n-1}{b^n(b-1)}$ (c) $an^3+\frac{b^{n-1}-1}{b^n(n-1)}$ (d) none of these

Q 92. If $s_n=\sum_{n=1}^{n}\frac{1+2+2^2+\ldots\text{to n terms}}{2^n}$ the s_n is equal to

(a) $2^n-(n+1)$ (b) $1-\frac{1}{2^n}$ (c) $n-1+\frac{1}{2^n}$ (d) 2^n-1

Q 93. Let S_n denote the sum of the cubes of the first n natural numbers s_n denote the sum of the first n natural numbers. Then $\sum_{r=1}^{n}\frac{S_r}{s_r}$ is equal to

(a) $\frac{n(n+1)(n+2)}{6}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n^2+3n+2}{6}$ (d) none of these

Q 94. It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$. Then $\sum_{r=1}^{\infty}\frac{1}{r^2}$ is equal to

(a) $\frac{\pi^2}{24}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) none of these

Q 95. It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\text{to }\infty=\frac{\pi^4}{90}$. Then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots\text{to }\infty$ is equal to

(a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^4}{45}$ (c) $\frac{89\pi^4}{90}$ (d) none of these

Q 96. If in a series $t_n=\frac{n}{(n+1)!}$ then $\sum_{n=1}^{20}t_n$ is equal to

(a) $\frac{20!-1}{20!}$ (b) $\frac{21!-1}{21!}$ (c) $\frac{1}{2(n-1)!}$ (d) none of these

Q 97. If t_n denotes the nth term of the series 2 + 3 + 6 + 11 + 18 + …. Then t_{50} is

(a) 49^2 -1 (b) $49^2 + 2$ (c) $50^2 + 1$ (d) $49^2 + 2$

Q 98. $2^{1/4} . 4^{1/8} . 8^{1/16}$ ... to ∞ is equal to

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) none of these

Q 99. The sum of n terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots\ldots$$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is

(a) $\frac{n^2(n+1)}{2}$ (b) $\frac{n^2(n-1)}{2}$ (c) $2(n + 1)^2. (2n + 1)$ (d) none of these

Q 100. If n is an odd integer greater than or equal to 1 then the value of $n^3 - (n-1)^3 + (n-2)^3 - \ldots. + (-1)^{n-1} . 1^3$ is

(a) $\frac{(n+1)^2.(2n-1)}{4}$ (b) $\frac{(n-1)^2.(2n-1)}{4}$ (c) $\frac{(n+1)^2.(2n+1)}{4}$ (d) none of these

Q 101. Observe that

$$1^3 = 1, 2^3 = 3 + 5, 3^3 = 7 + 9 + 11, 4^3 = 13 + 15 + 17 + 19.$$

Then n^3 as a similar series is

(a) $\left[2\left\{\frac{n(n-1)}{2}+1\right\}-1\right]+\left[2\left\{\frac{(n+1)n}{2}+1\right\}+1\right]+\ldots\ldots+\left[2\left\{\frac{(n+1)n}{2}+1\right\}+2n-3\right]$

(b) $(n^2 + n + 1) + (n^2 + n + 3) + (n^2 + n + 5) + \ldots.. + (n^2 + 3n - 1)$

(c) $(n^2 - n + 1) + (n^2 - n + 3) + (n^2 - n + 5) + \ldots.. + (n^2 + n - 1)$

(d) none of these

Q 102. Let $t_r = 2^{r/2} + 2^{-r/2}$. Then $\sum_{r=1}^{10} t_r^2$ is equal to

(a) $\frac{2^{21}-1}{2^{10}}+20$ (b) $\frac{2^{21}-1}{2^{10}}+19$ (c) $\frac{2^{21}-1}{2^{20}}-1$ (d) none of these

Q 103. Let $S_k = \lim_{n\to\infty}\sum_{i=0}^{n}\frac{1}{(k+1)^i}$. Then $\sum_{k=1}^{n} kS_k$ equals

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{n(n+2)}{2}$ (d) $\frac{n(n+3)}{2}$

Q 104. Let $t_n = n.(n!)$. then $\sum_{n=1}^{15} t_n$ is equal to

(a) $15! - 1$ (b) $15! + 1$ (c) $16! - 1$ (d) none of these

Q 105. The sum of $\frac{3}{1.2}.\frac{1}{2}+\frac{4}{2.3}.\left(\frac{1}{2}\right)^2+\frac{5}{3.4}.\left(\frac{1}{2}\right)^3+.....$to n terms is equal to

(a) $1-\frac{1}{(n+1)2^n}$ (b) $1-\frac{1}{n.2^{n-1}}$ (c) $1+\frac{1}{(n+1)2^n}$ (d) none of these

Q 106. Let $f(n) = \left[\frac{1}{2}+\frac{n}{100}\right]$ where [x] denotes the integral part of x. Then the value of $\sum_{n=1}^{100} f(n)$ is

(a) 50 (b) 51 (c) 1 (d) none of these

Q 107. A_r; r = 1, 2, 3,....., n are n points on the parabola $y^2 = 4x$ in the first quadrant. If $A_r = (x_r, y_r)$, where $x_1, x_2, x_3,.....,x_n$ are in GP and $x_1 = 1$, $x_2 = 2$, then y_n is equal to

(a) $-2^{\frac{n+1}{2}}$ (b) 2^{n+1} (c) $(\sqrt{2})^{n+1}$ (d) $2^{\frac{n}{2}}$

Q 108. In the given square, a diagonal is drawn, and parallel line the segments joining points on the adjacent sides are drawn on both sides of the diagonal. The length of the diagonal $n\sqrt{2}$ cm. If the distance between consecutive line segment be $1/\sqrt{2}$ cm then the sum of the lengths of all possible line segments and the diagonal is

(a) $n(n + 1)\sqrt{2}$ cm (b) n^2cm (c) $n(n+2)$cm (d) $n^2\sqrt{2}$cm

Q 109. ABCD is a square of length a, a $\in$ N, a > 1. Let L_1, L_2, L_3, ... be points on BC such that $BL_1 = L_1L_2 = L_2L_3 = ... = 1$ and M_1, M_2, M_3,.... be points on CD such that $CM_1 = M_1M_2 = M_2M_3 = ... = 1$. Then $\sum_{n=1}^{a-1}(AL_n^2 + L_nM_n^2)$ is equal to

(a) $\frac{1}{2}a(a-1)^2$ (b) $\frac{1}{2}a(a-1)(4a-1)$ (c) $\frac{1}{2}(a-1)(2a-1)(4a-1)$ (d) none of these

Q 110. The sum of infinite terms of a decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval [-2, 3] and the difference between the first two terms is f'(0). Then the common ratio of the GP is

(a) $-\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

Q 111. The lengths of three unequal edges of a rectangular solid block are in GP. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. The length of the longest edge is

(a) 12 cm (b) 6 cm (c) 18 cm (d) 3 cm

Q 112. ABC is a right-angled triangle in which $\angle B = 90^\circ$ and BC = a. If n points $L_1, L_2, \ldots, L_n$ on AB are such that AB is divided in n + 1 equal parts and $L_1M_1, L_2M_2, \ldots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, \ldots, M_n$ are on AC then the sum of the lengths of $L_1M_1, L_2M_2, \ldots, L_nM_n$ is

(a) $\frac{a(n+1)}{2}$ (b) $\frac{a(n-1)}{2}$ (c) $\frac{an}{2}$

(d) impossible to find from the given data

Choose the correct options. One or more options may be correct.

Q 113. If AM of the number 5^{1+x} and 5^{1-x} is 13 then the set of possible real values of x is

(a) $\left\{5, \frac{1}{5}\right\}$ (b) {1, 1} (c) $\{x \mid x^2 - 1 = 0, x \in R\}$ (d) none of these

Q 114. If the AM of two positive numbers be three times their geometric mean then the ratio of the numbers is

(a) $3 \pm 2\sqrt{2}$ (b) $\sqrt{2} \pm 1$ (c) $17 + 12\sqrt{2}$ (d) $(3-2\sqrt{2})^{-2}$

Q 115. If a, b, c are in HP then $\frac{1}{b-a} + \frac{1}{b-c}$ is equal to

(a) $\frac{2}{b}$ (b) $\frac{2}{a+c}$ (c) $\frac{1}{a} + \frac{1}{c}$ (d) none of these

Q 116. S_r denotes the sum of the first r terms of an AP. Then $S_{3n} : (S_{2n} - S_n)$ is

(a) n (b) 3n (c) 3 (d) independent of n

Q 117. If $a^x = b^y = c^z$ and x, y, z are in GP then $\log_c b$ is equal to

(a) $\log_b a$ (b) $\log_a b$ (c) $\frac{z}{y}$ (d) none of these

Q 118. The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$ is

(a) $\frac{n}{\sqrt{a} + \sqrt{a+nx}}$ (b) $\frac{\sqrt{a+nx} - \sqrt{a}}{x}$ (c) $\frac{n(\sqrt{a+nx} - a)}{x}$ (d) none of these

Q 119. Let $\sum_{n=1}^{n} r^4 = f(n)$. Then $\sum_{r=1}^{n} (2r-1)^4$ is equal to

(a) $f(2n) - 16f(n)$ for all $n \in N$

(b) $f(n) - 16f\left(\frac{n-1}{2}\right)$ when n is odd

(c) $f(n) - 16f\left(\frac{n}{2}\right)$ when n is even

(d) none of these

Q 120. If $2.{}^nP_1, {}^nP_2, {}^nP_3$ are three consecutive terms of an AP then they are

(a) in GP (b) in HP (c) equal (d) none of these

Q 121. In a GP the product of the first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of the GP up to infinite terms is

(a) 8 (b) -8 (c) $\frac{8}{3}$ (d) $-\frac{8}{3}$

Q 122. If $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$ then

(a) $a = \frac{1}{12}$ (b) $b = \frac{1}{6}$ (c) $d = \frac{1}{6}$ (d) $e = 0$

Q 123. If a, b, c, d are four positive numbers then

(a) $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \geq 4.\sqrt{\frac{a}{e}}$

(b) $\left(\frac{a}{b}+\frac{c}{d}\right)\left(\frac{b}{c}+\frac{d}{e}\right) \geq 4.\sqrt{\frac{a}{e}}$

(c) $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a} \geq 5$

(d) $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \geq \frac{1}{5}$

Q 124. Let $f(x) = \frac{1-x^{n+1}}{1-x}$ and $g(x) = 1 - \frac{2}{x} + \frac{3}{x^2} - .... + (-1)^n \frac{n+1}{x^n}$. Then the constant term in f'(x) × g(x) is equal to

(a) $\frac{n(n^2-1)}{6}$ when n is even

(b) $\frac{n(n+1)}{2}$ when n is odd

(c) $-\frac{n}{2}(n+1)$ when n is even

(d) $-\frac{n(n-1)}{2}$ when n is odd

Q 125. Let a_n = product of the first n natural numbers. Then for all $n \in N$

(a) $a^n \geq a_n$ (b) $\left(\frac{n+1}{2}\right)^n \geq n!$ (c) $n^n \geq a_{n+1}$ (d) none of these

Q 126. Let the sets A = {2, 4, 6, 8,....} and B = {3, 6, 9, 12,....} and n(A) = 200, n(B) = 250. Then

(a) $n(A \cap B) = 67$ (b) $n(A \cup B) = 450$ (c) $n(A \cap B) = 66$ (d) $n(A \cup B) = 384$

Q 127. Let a, x, b be in AP; a, y, b be in GP and a, z, b be in HP. If x = y + 2 and a = 5z then

(a) $y^2 = xz$ (b) $x > y > z$ (c) a = 9, b = 1 (d) $a = \frac{1}{4}, b = \frac{9}{4}$

Q 128. Let $S_1, S_2, S_3, \ldots$.be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm then for which of the following values of n is the area of S_n less than 1 cm^2 ?

(a) 7 (b) 8 (c) 9 (d) 10

Q 129. Three positive numbers from a GP. If the middle number is increased by 8, the three numbers form an AP. If the last number is also increased by 64 along with the previous increase in the middle number, the resulting numbers from a GP again. Then

(a) common ratio = 3 (b) first number = $\frac{4}{9}$ (c) common ratio = -5 (d) first number = 4

Q 130. If a, b, c are in GP and a, p, q are in AP such that 2a, b + p, c + q are in GP then the common difference of the AP is

(a) $\sqrt{2}a$ (b) $(\sqrt{2}+1)(a-b)$ (c) $\sqrt{2}(a+b)$ (d) $(\sqrt{2}-1)(b-a)$

Q 131. If x, y, z are positive numbers in AP then

(a) $y^2 \geq xy$ (b) $y \geq 2\sqrt{xz}$ (c) $\frac{x+y}{2y-x} + \frac{y+z}{2y-z}$ has the minimum value 2

(d) $\frac{x+y}{2y-x} + \frac{y+z}{2y-z} \geq 4$

Q 132. Between two unequal numbers, if a_1, a_2 are two AMs; g_1, g_2 are two GMs and h_1, h_2 are two HMs then $g_1 . g_2$ is equal to

(a) a_1h_1 (b) a_1h_2 (c) a_2h_2 (d) a_2h_1

Q 133. The number 1, 4, 16 can be three terms (not necessarily consecutive) of

(a) no AP (b) only one GP

(c) infinite number of APs (d) infinite number of GPs

Answers

1a	2c	3b	4c	5c	6d	7c	8b	9a	10c
11c	12b	13c	14d	15b	16a	17b	18a	19c	20b
21d	22a	23b	24d	25c	26a	27a	28b	29a	30d
31a	32b	33c	34a	35b	36c	37b	38a	39c	40b
41b	42c	43b	44b	45b	46d	47c	48b	49b	50b
51d	52b	53c	54b	55a	56c	57a	58a	59a	60a
61c	62a	63d	64a	65a	66b	67a	68c	69c	70b
71c	72b	73c	74d	75d	76b	77c	78b	79a	80b
81c	82c	83b	84a	85c	86a	87c	88b	89d	90c
91b	92c	93a	94c	95a	96b	97d	98a	99a	100a
101c	102b	103d	104c	105a	106b	107c	108d	109b	110c
111a	112c	113bc	114cd	115ac	116cd	117ac	118ab	119a	120abc
121abcd	122ac	123abc	124bc	125ab	126cd	127ac	128bcd	129ad	130bd
131ad	132bd	133cd							

Equation, Inequation and Expression

Choose the most appropriate option (a, b, c or d).

Q 1. If x is a real number such that $x(x^2 + 1)$, $(-1/2)x^2$, 6 are three consecutive terms of an AP then the next two consecutive term of the AP are

(a) 14, 6 (b) -2, -10 (c) 14, 22 (d) none of these

Q 2. The number of real solutions of $x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$ is

(a) 0 (b) 1 (c) 2 (d) infinite

Q 3. The number of values of a for which

$$(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 + a^2 - 4 = 0$$

is an identity in x if

(a) 0 (b) 2 (c) 1 (d) 3

Q 4. The number of values of the pair (a, b) for which

$$a(x + 1)^2 + b(x^2 - 3x - 2) + x + 1 = 0$$

is an identity in x is

(a) 0 (b) 1 (c) 2 (d) infinite

Q 5. The number of values of the triplet (a, b, c) for which

$$a \cos 2x + b \sin^2 x + c = 0$$

is satisfied by all real x is

(a) 0 (b) 2 (c) 3 (d) infinite

Q 6. The polynomial $(ax^2 + bx + c)(ax^2 - dx - c)$, $ac \neq 0$, has

(a) four real zeros (b) at least two real zeros

(c) at most two real zeros (d) no real zeros

Q 7. Let $f(x) = ax^3 + 5x^2 - bx + 1$. If f(x) when divided by 2x + 1 leaves 5 as remainder, and f'(x) is divisible by 3x – 1 then

(a) a = 26, b = 10 (b) a = 24, b = 12 (c) a = 26, b = 12 (d) none of these

Q 8. $x^{3^n} + y^{3^n}$ is divisible by x + y if

(a) n is any integer ≥ 0 (b) n is an odd positive integer

(c) n is an even positive integer (d) n is a rational number

Q 9. If x, y are rational numbers such that

$$x + y + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$$

then

(a) x and y cannot be determined (b) $x = 2, y = 1$

(c) $x = 5, y = 1$ (d) none of these

Q 10. The number of real solutions of the equation

$$2^{x/2} + (\sqrt{2} + 1)^x = (5 + 2\sqrt{2})^{x/2}$$

is

(a) one (b) two (c) four (d) infinite

Q 11. The number of real solutions of the equation $e^x = x$ is

(a) 1 (b) 2 (c) 0 (d) none of these

Q 12. The sum of the real roots of the equation $x^2 + |x| - 6 = 0$ is

(a) 4 (b) 0 (c) -1 (d) none of these

Q 13. The solutions of the equation $2x - 2[x] = 1$, where [x] = the greatest integer less than or equal to x, are

(a) $x = n + \frac{1}{2}, n \in N$ (b) $x = n - \frac{1}{2}, n \in N$ (c) $x = n + \frac{1}{2}, n \in Z$ (d) $n < x < n + 1, n \in Z$

Q 14. The number of real solutions of the equation $\sin(e^x) = 5^x + 5^{-x}$ is

(a) 0 (b) 1 (c) 2 (d) infinitely many

Q 15. The number of real solution of $1 + |e^x - 1| = e^x(e^x - 2)$ is

(a) 0 (b) 1 (c) 2 (d) 4

Q 16. The equation $2\sin^2\frac{x}{2}.\cos^2 x = x + \frac{1}{x}, 0 < x \le \frac{\pi}{2}$ has

(a) one real solution (b) no real solution

(c) infinitely many real solutions (d) none of these

Q 17. If $y \neq 0$ then the number of values of the pair (x, y) such that

$$x + y + \frac{x}{y} = \frac{1}{2} \text{ and } (x + y)\frac{x}{y} = -\frac{1}{2}, \text{ is}$$

(a) 1 (b) 2 (c) 0 (d) none of these

Q 18. The number of real solutions of the equation $\log_{0.5} x = |x|$ is

(a) 1 (b) 2 (c) 0 (d) none of these

Q 19. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has

(a) no solution (b) one solution (c) two solutions (d) more than two solutions

Q 20. The number of solutions of the equation $|x| = \cos x$ is

(a) one (b) two (c) three (d) zero

Q 21. The product of all the solutions of the equation

$$(x - 2)^2 - 3\,|x - 2| + 2 = 0$$

is

(a) 2 (b) -4 (c) 0 (d) none of these

Q 22. If $0 < x < 1000$ and $\left[\frac{x}{2}\right] + \left[\frac{x}{3}\right] + \left[\frac{x}{5}\right] = \frac{31}{30}x$, where [x] is the greatest integer less than or equal to x, the number of possible values of x is

(a) 34 (b) 32 (c) 33 (d) none of these

Q 23. The solution set of $(x)^2 + (x + 1)^2 = 25$, where (x) is the least integer greater than or equal to x, is

(a) (2, 4) (b) $(-5, -4] \cup (2, 3]$ (c) $[-4, -3) \cup [3, 4)$ (d) none of these

Q 24. If $3^{x+1} = 6^{\log_2 3}$ then x is

(a) 3 (b) 2 (c) $\log_3 2$ (d) $\log_2 3$

Q 25. If $(\sqrt{2})^x + (\sqrt{3})^x = (\sqrt{13})^{x/2}$ then the number of values of x is

(a) 2 (b) 4 (c) 1 (d) none of these

Q 26. The number of real solutions of the equation $\frac{6-x}{x^2-4} = 2 + \frac{x}{x+2}$ is

(a) two (b) one (c) zero (d) none of these

Q 27. The number of real solutions of

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$ is

(a) one (b) two (c) three (d) none of these

Q 28. If [x] = the greatest integer less than or equal to x, and (x) = the least integer greatest than or equal to x, and $[x]^2 + (x)^2 > 25$ then x belongs to

(a) [3, 4] (b) $(-\infty, -4]$ (c) $[4, +\infty)$ (d) $(-\infty, -4] \cup [4, +\infty)$

Q 29. Let R = the set of real numbers, $\mathbb{Z}$ = the set of integers, N = the set of natural numbers. If S be the solution set of the equation $(x)^2 + [x]^2 = (x-1)^2 + [x+1]^2$, where (x) = the least integer greater than or equal to x and [x] = the greatest integer less than or equal to x, then

(a) S = R (b) S = R - $\mathbb{Z}$ (c) S = R – N (d) none of these

Q 30. If $[x]^2 = [x + 2]$, where [x] = the greatest integer less than or equal to x, then x must be such that

(a) x = 2, - 1 (b) $x \in [2, 3)$ (c) $x \in [-1, 0)$ (d) none of these

Q 31. The solution set of $\left|\frac{x+1}{x}\right| + |x + 1| = \frac{(x+1)^2}{|x|}$ is

(a) $\{x \mid x \geq 0\}$ (b) $\{x \mid x > 0\} \cup \{-1\}$ (c) {-1, 1} (d) $\{x \mid x \geq 1 \text{ or } x \leq -1\}$

Q 32. The number of solutions of $|[x] - 2x| = 4$, where [x] is the greatest integer $\leq x$, is

(a) 2 (b) 4 (c) 1 (d) infinite

Q 33. The set of real values of x satisfying $|x - 1| \leq 3$ and $|x - 1| \geq 1$ is

(a) [2, 4] (b) $(-\infty, 2] \cup [4, +\infty)$ (c) $[-2, 0] \cup [2, 4]$ (d) none of these

Q 34. The set of real values of x satisfying $||x-1|-1| \leq 1$ is

(a) [-1, 3] (b) [0, 2] (c) [-1, 1] (d) none of these

Q 35. If $x \in \mathbb{Z}$ (the set of integers) such that $x^2 - 3x < 4$ then the number of possible values of x is

(a) 3 (b) 4 (c) 6 (d) none of these

Q 36. If x is an integer satisfying $x^2 - 6x + 5 \leq 0$ and $x^2 - 2x > 0$ then the number of possible values of x is

(a) 3 (b) 4 (c) 2 (d) infinite

Q 37. The solution set of the inequation $\log_{1/3}(x^2 + x + 1) + 1 > 0$ is

(a) $(-\infty, -2) \cup (1, +\infty)$ (b) [-1, 2] (c) (-2, 1) (d) $(-\infty, +\infty)$

Q 38. If $5^x + (2\sqrt{3})^{2x} \geq 13^x$ then the solution set for x is

(a) $[2, +\infty)$ (b) $\{2\}$ (c) $(-\infty, 2]$ (d) $[0, 2]$

Q 39. If $3^{x/2} + 2^x > 25$ then the solution set is

(a) R (b) $(2, +\infty)$ (c) $(4, +\infty)$ (d) none of these

Q 40. If $\sin^x \alpha + \cos^x \alpha \geq 1, 0 < \alpha < \frac{\pi}{2}$, then

(a) $x \in [2, +\infty)$ (b) $x \in (-\infty, 2)$ (c) $x \in [-1, 1]$ (d) none of these

Q 41. The solution set of $x^2 + 2 \leq 3x \leq 2x^2 - 5$ is

(a) ϕ (b) $[1, 2]$ (c) $(-\infty, -1] \cup [5/2, +\infty)$ (d) none of these

Q 42. The solution set of $\frac{x^2 - 3x + 4}{x + 1} > 1, x \in R$, is

(a) $(3, +\infty)$ (b) $(-1, 1) \cup (3, +\infty)$ (c) $[-1, 1] \cup [3, +\infty)$ (d) none of these

Q 43. The number of integral solutions of $\frac{x + 2}{x^2 + 1} > \frac{1}{2}$ is

(a) 4 (b) 5 (c) 3 (d) none of these

Q 44. If a, b, c are nonzero, unequal rational numbers then the roots of the equation $abc^2x^2 + (3a^2 + b^2)cx - 6a^2 - ab + 2b^2 = 0$ are

(a) rational (b) imaginary (c) irrational (d) none of these

Q 45. If l, m are real and $l \neq m$ then the roots of the equation

$(l - m)x^2 - 5(l + m)x - 2(l - m) = 0$ are

(a) real and equal (b) nonreal complex (c) real and unequal (d) none of these

Q 46. If a, b, c, d are four consecutive terms of an increasing AP then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(a) real and distinct (b) nonreal complex (c) real and equal (d) integers

Q 47. If a, b, c are three distinct positive real number then the number of real roots of $ax^2 + 2b|x| - c = 0$ is

(a) 4 (b) 2 (c) 0 (d) none of these

Q 48. The equation $x^2 - 6x + 8 + \lambda(x^2 - 4x + 3) = 0, \lambda \in R$, has

(a) real and unequal roots for all λ (b) real roots for $\lambda < 0$ only

(c) real roots for $\lambda > 0$ only (d) real and unequal roots for $\lambda = 0$ only

Q 49. If $\cos\theta$, $\sin\phi$, $\sin\theta$ are in GP then roots of $x^2 + 2\cot\phi \,.\, x + 1 = 0$ are always

(a) equal (b) real (c) imaginary (d) greater than 1

Q 50. The roots of $ax^2 + bx + c = 0$, where $a \neq 0$ and coefficients are real, are nonreal complex and $a + c < b$. Then

(a) $4a + c > 2b$ (b) $4a + c < 2b$ (c) $4a + c = 2b$ (d) none of these

Q 51. The equation $(a + 2)x^2 + (a - 3)x = 2a - 1$, $a \neq -2$ has roots rational for

(a) all rational values of except $a = -2$ (b) all real values of a except $a = -2$

(c) rational values of $a > \frac{1}{2}$ (d) none of these

Q 52. If $a \,.\, 3^{\tan x} + a \,.\, 3^{-\tan x} - 2 = 0$ has real solutions, $x \neq \frac{\pi}{2}$, $0 \leq x \leq \pi$, then the set of possible values of the parameter a is

(a) $[-1, 1]$ (b) $[-1, 0)$ (c) $(0, 1]$ (d) $(0, +\infty)$

Q 53. If $a > 1$, roots of the equation $(1 - a)x^2 + 3ax - 1 = 0$ are

(a) one positive and one negative (b) both negative

(c) both positive (d) both nonreal complex

Q 54. If $a \in R$, $b \in R$ then the equation $x^2 - abx - a^2 = 0$ has

(a) one positive root and one negative root (b) both roots positive

(c) both roots negative (d) nonreal roots

Q 55. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are less than 3 then

(a) $a < 2$ (b) $2 \leq a \leq 3$ (c) $3 < a \leq 4$ (d) $a > 4$

Q 56. If α, β are the roots of $x^2 - 3x + a = 0$, $a \in R$ and $\alpha < 1 < \beta$ then

(a) $a \in (-\infty, 2)$ (b) $a \in \left(-\infty, \frac{9}{4}\right]$ (c) $a \in \left(2, \frac{9}{4}\right]$ (d) none of these

Q 57. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, $\lambda \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$ then the number of integral solutions of λ is

(a) 5 (b) 6 (c) 2 (d) 3

Q 58. The number of integer values of a for which $x^2 - (a-1)x + 3 = 0$ has both roots positive and $x^2 + 3x + 6 - a = 0$ has both roots negative is

(a) 0 (b) 1 (c) 2 (d) infinite

Q 59. If X denotes the set of real numbers p for which the equation $x^2 = p(x + p)$ has its roots greater than p then X is equal to

(a) $\left(-2, -\frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{1}{4}\right)$ (c) null set ϕ (d) $(-\infty, 0)$

Q 60. If $\cos^4 x + \sin^2 x - p = 0$, $p \in R$ has real solutions then

(a) $p \le 1$ (b) $\frac{3}{4} \le p \le 1$ (c) $p \ge \frac{3}{4}$ (d) none of these

Q 61. If one root of the equation $(k^2 + 1)x^2 + 13x + 4k = 0$ is reciprocal of the other then k has the value

(a) $-2 + \sqrt{3}$ (b) $2 - \sqrt{3}$ (c) 1 (d) none of these

Q 62. If the ratio of the roots of $\lambda x^2 + \mu x + \nu = 0$ is equal to the ratio of the roots of $x^2 + x + 1 = 0$ then λ, μ, ν are in

(a) AP (b) GP (c) HP (d) none of these

Q 63. p, q, r and s are integers. If the AM of the roots of $x^2 - px + q^2 = 0$ and GM of the roots of $x^2 - rx + s^2 = 0$ are equal then

(a) q is an odd integer (b) r is an even integer (c) p is an even integer (d) s is an odd integer

Q 64. If α, β are roots of the equation $(x - a)(x - b) = c$, $c \ne 0$, then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are

(a) a, c (b) b, c (c) a, b (d) a + c, b + c

Q 65. If the roots of $4x^2 + 5k = (5 + 1)x$ differ by unity then the negative value of k is

(a) -3 (b) $-\frac{1}{5}$ (c) $-\frac{3}{5}$ (d) none of these

Q 66. The harmonic mean of the roots of the equation

$$(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0 \text{ is}$$

(a) 2 (b) 4 (c) 6 (d) 8

Q 67. If the product of the roots of the equation $x^2 - 5x + 4^{\log_2 \lambda} = 0$ is 8 then λ is

(a) $\pm 2\sqrt{2}$ (b) $2\sqrt{2}$ (c) 3 (d) none of these

Q 68. If the roots of $a_1x^2 + b_1x + c_1 = 0$ are α_1, β_1, and those of

$a_2x^2 + b_2x + c_2 = 0$ are α_2, β_2 such that $\alpha_1\alpha_2 = \beta_1\beta_2 = 1$

then

(a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (b) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$ (c) $a_1a_2 = b_1b_2 = c_1c_2$ (d) none of these

Q 69. If α, β are the roots of $ax^2 + c = bx$ then the equation $(a + cy)^2 = b^2y$ in y has the roots

(a) α^{-1}, β^{-1} (b) α^2, β^2 (c) $\alpha\beta^{-1}, \alpha^{-1}\beta$ (d) α^{-2}, β^{-2}

Q 70. If the roots of $ax^2 - bx - c = 0$ change by the same quantity then the expression in a, b, c that does not change is

(a) $\frac{b^2 - 4ac}{a^2}$ (b) $\frac{b - 4c}{a}$ (c) $\frac{b^2 + 4ac}{a^2}$ (d) none of these

Q 71. If α, β are the roots of $x^2 - px + q = 0$ then the product of the roots of the quadratic equation whose roots are $\alpha^2 - \beta^2$ and $\alpha^3 - \beta^3$ is

(a) $p(p^2 - q)^2$ (b) $p(p^2 - q)(p^2 - 4q)$ (c) $p(p^2 - 4q)(p^2 + q)$ (d) none of these

Q 72. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\frac{b^2}{ac} + \frac{bc}{a^2}$ is equal to

(a) 2 (b) -2 (c) 1 (d) -1

Q 73. If the absolute value of the difference of roots of the equation $x^2 + px + 1 = 0$ exceed $\sqrt{3}p$ then

(a) $p < -1$ or $p > 4$ (b) $p > 4$ (c) $-1 < p < 4$ (d) $0 \le p < 4$

Q 74. If α, β are roots of $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$ then $(\alpha - \gamma)(\alpha - \delta)$ is equal to

(a) $q + r$ (b) $q - r$ (c) $-(q + r)$ (d) $-(p + q + r)$

Q 75. If α, β are roots of $375x^2 - 25x - 2 = 0$ and $s_n = \alpha^n + \beta^n$ then $\lim_{n\to\infty}\sum_{r=1}^{n} s_r$ is

(a) $\frac{7}{116}$ (b) $\frac{1}{12}$ (c) $\frac{29}{358}$ (d) none of these

Q 76. The quadratic equation whose roots are the AM and HM of the roots of the equation $x^2 + 7x - 1 = 0$ is

(a) $14x^2 + 14x - 45 = 0$ (b) $45x^2 - 14x + 14 = 0$

(c) $14x^2 + 45x - 14 = 0$ (d) none of these

Q 77. Let $\alpha \neq \beta$ and $\alpha^2 + 3 = 5\alpha$ while $\beta^2 = 5\beta - 3$. The quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

(a) $3x^2 - 31x + 3 = 0$ (b) $3x^2 - 19x + 3 = 0$ (c) $3x^2 + 19x + 3 = 0$ (d) none of these

Q 78. If a and b are rational and b is not a perfect square then the quadratic equation with rational coefficients whose one root is $\frac{1}{a+\sqrt{b}}$ is

(a) $x^2 - 2ax + (a^2 - b) = 0$ (b) $(a^2 - b)x^2 - 2ax + 1 = 0$

(c) $(a^2 - b)x^2 - 2bx + 1 = 0$ (d) none of these

Q 79. If $\frac{1}{4-3i}$ is a root of $ax^2 + bx + 1 = 0$, where a, b are real, then

(a) a = 25, b = -8 (b) a = 25, b = 8 (c) a = 5, b = 4 (d) none of these

Q 80. If α, β, γ be the roots of the equation $x(1 + x^2) + x^2(6 + x) + 2 = 0$ then the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ is

(a) -3 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) none of these

Q 81. If the roots of $x^3 - 12x^2 + 39x - 28 = 0$ are in AP then their common difference is

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

Q 82. The roots of the equation $x^3 + 14x^2 - 84x - 216 = 0$ are in

(a) AP (b) GP (c) HP (d) none of these

Q 83. If $z_0 = \alpha + i\beta$, = $\sqrt{-1}$, then the roots of the cubic equation

$$x^3 - 2(1 + \alpha)x^2 + (4\alpha + \alpha^2 + \beta^2)x - 2(\alpha^2 + \beta^2) = 0 \text{ are}$$

(a) $2, z_0, \bar{z}_0$ (b) $1, z_0, -z_0$ (c) $2, z_0, -\bar{z}_0$ (d) $2, -z_0, \bar{z}_0$

Q 84. If 3 and $1 + \sqrt{2}$ are two roots of a cubic equation with rational coefficients then the equation is

(a) $x^2 - 5x^2 + 9x - 9 = 0$ (b) $x^3 - 3x^2 - 4x + 12 = 0$ (c) $x^3 - 5x^2 + 7x + 3 = 0$ (d) none of these

Q 85. Let a, b, c be real numbers and $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$, and $0 < \alpha < \beta$ then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(a) $\gamma = \frac{1}{2}(\alpha + \beta)$ (b) $\gamma = \alpha + \frac{\beta}{2}$ (c) $\gamma = \alpha$ (d) $\alpha < \gamma < \beta$

Q 86. Let a, b, c three real number such that $2a + 3b + 6c = 0$. Then the quadratic equation $ax^2 + bx + c = 0$ has

(a) imaginary roots (b) at least one root in (0, 1)

(c) at least one root in (-1, 0) (d) both roots in (1, 2)

Q 87. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root then

(a) a = 2, b = -7 (b) $a = -\frac{7}{2}, b = 1$ (c) a = 4, b = -14 (d) none of these

Q 88. The quadratic equations $x^2 + (a^2 - 2)x - 2a^2 = 0$ and $x^2 - 3x + 2 = 0$ have

(a) no common root for all $a \in R$ (b) exactly one common root for all $a \in R$

(c) two common roots for some $a \in R$ (d) none of these

Q 89. If the equation $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$, $a \neq c$ have a negative common root then the value of $a - b + c$ is

(a) 0 (b) 2 (c) 1 (d) none of these

Q 90. If the equations $x^2 + ix + a = 0$, $x^2 - 2x + ia = 0$, $a \neq 0$ have a common root then

(a) a is real (b) $a = \frac{1}{2} + i$ (c) $a = \frac{1}{2} - i$ (d) the other root is also common

Q 91. If $x^2 - 2r \cdot p_r x + r = 0$; r = 1, 2, 3 are three quadratic equations of which each pair has exactly one root common then the number of solutions of the triplet (p_1, p_2, p_3) is

(a) 2 (b) 1 (c) 9 (d) 27

Q 92. If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$ then λ belongs to the interval

(a) (-2, 1) (b) $\left(-2, \frac{2}{5}\right)$ (c) $\left(\frac{2}{5}, 1\right)$ (d) none of these

Q 93. The least integral value of k for which $(k - 2)x^2 + 8x + k + 4 > 0$ for all $x \in R$, is

(a) 5 (b) 4 (c) 3 (d) none of these

Q 94. The set of possible values of x such that $5^x + (2\sqrt{3})^{2x} - 169$ is always positive is

(a) $[3, +\infty)$ (b) $[2, +\infty)$ (c) $(2, +\infty)$ (d) none of these

Q 95. If all real values of x obtained from the equation

$$4^x - (a-3)2^x + a - 4 = 0$$

are nonpositive then

(a) $a \in (4, 5]$ (b) $a \in (0, 4)$ (c) $a \in (4, +\infty)$ (d) none of these

Q 96. The set of possible values of λ for which

$$x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$$

has roots whose sum and product are both less than 1 is

(a) $\left(1, \frac{5}{2}\right)$ (b) $(1, 4)$ (c) $\left[1, \frac{5}{2}\right]$ (d) $\left(1, \frac{5}{2}\right)$

Q 97. If $\log_{10} x + \log_{10} y \geq 2$ then the smallest possible value of x + y is

(a) 10 (b) 30 (c) 20 (d) none of these

Q 98. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$ for every real number x then the minimum value of f

(a) does not exist because f is unbounded (b) is not attained even though f is bounded

(c) is equal to 1 (d) is equal to -1

Q 99. If $ax^2 + bx + 6 = 0$ does not have two distinct real roots, where $a \in R$, $b \in R$, then the least value of 3a + b is

(a) 4 (b) -1 (c) 1 (d) -2

Q 100. If ab = 2a + 3b, a > 0, b > 0 then the minimum value of ab is

(a) 12 (b) 24 (c) $\frac{1}{4}$ (d) none of these

Q 101. If $x^2 + px + 1$ is a factor of the expression $ax^3 + bx + c$ then

(a) $a^2 + c^2 = -ab$ (b) $a^2 - c^2 = -ab$ (c) $a^2 - c^2 = ab$ (d) none of these

Q 102. If $x^2 - 1$ is a factor of $x^4 + ax^3 + 3x - b$ then

(a) a = 3, b = -1 (b) a = -3, b = 1 (c) a = 3, b = 1 (d) none of these

Q 103. The number of values of k for which

$$\{x^2 - (k-2)x + k^2\}\{x^2 + kx + (2k-1)\}$$

is a perfect square is

(a) 1 (b) 2 (c) 0 (d) none of these

Q 104. If $x+ \lambda y - 2$ and $x - \mu y + 1$ are factors of the expression

$$6x^2 - xy - y^2 - 6x + 8y - 12$$

then

(a) $\lambda = \frac{1}{3}, \mu = \frac{1}{2}$ (b) $\lambda = 2, \mu = 3$ (c) $\lambda = \frac{1}{3}, \mu = -\frac{1}{2}$ (d) none of these

Q 105. If $x - y$ and $y - 2x$ are two factors of the expression

$$x^3 - 3x^2y + \lambda xy^2 + \mu y^3$$

then

(a) $\lambda = 11, \mu = -3$ (b) $\lambda = 3, \mu = -11$ (c) $\lambda = \frac{11}{4}, \mu = -\frac{3}{4}$ (d) none of these

Q 106. If $x + y$ and $y + 3x$ are two factors of the expression

$$\lambda x^3 - \mu x^2y + xy^2 + y^3$$

then the third factor is

(a) $y + 3x$ (b) $y - 3x$ (c) $y - x$ (d) none of these

Q 107. If x, y, z are real and distinct then

$$f(x, y) = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$$

is always

(a) non-negative (b) nonpositive (c) zero (d) none of these

Q 108. If $x^2 + y^2 + z^2 = 1$ then the value of $xy + yz + zx$ lies in the interval

(a) $\left[\frac{1}{2}, 2\right]$ (b) [-1, 2] (c) $\left[-\frac{1}{2}, 1\right]$ (d) $\left[-1, \frac{1}{2}\right]$

Q 109. If $a \in R$, $b \in R$ then the factors of the expression $a(x^2 - y^2) - bxy$ are

(a) real and different (b) real and identical (c) complex (d) none of these

Q 110. If a, b, c are in HP then the expresson

$$a(b - c)x^2 + b(c - a)x + c(a - b)$$

(a) has real and distinct factors (b) is a perfect square

(c) has no real factor (d) none of these

Q 111. The number of positive integral values of k for which $(16x^2 + 12x + 39) + k(9x^2 - 2x + 11)$ is a perfect square is

(a) two (b) zero (c) one (d) none of these

Q 112. If $(x - 1)^3$ is a factor of $x^4 + ax^3 + bx^2 + cx - 1$ then the other factor is

(a) $x - 3$ (b) $x + 1$ (c) $x + 2$ (d) none of these

Choose the correct options. One or more options may be correct.

Q 113. If $x^2 - bx + c = 0$ has equal integral roots then

(a) b and c are integers (b) b and c are even integers

(c) b is an even integer and c is a perfect square of a positive integer

(d) none of these

Q 114. Let A, G and H be the AM, GM and HM of two positive numbers a and b. The quadratic equatin whose roots are A and H is

(a) $Ax^2 - (A^2 + G^2)x + AG^2 = 0$ (b) $Ax^2 - (A^2 + H^2)x + AH^2 = 0$

(c) $Hx^2 - (H^2 + G^2)x + HG^2 = 0$ (d) none of these

Q 115. Let A, G and H are the AM, GM and HM respectively of two unequal positive integers. Then the equation $Ax^2 - |G|x - H = 0$ has

(a) both roots as fractions (b) at least one root which is a negative fraction

(c) exactly one positive root (d) at least one root which is an integer

Q 116. Let $x^2 - px + q = 0$, where $p \in R$, $q \in R$, have the roots α, β such that $\alpha + 2\beta = 0$ then

(a) $2p^2 + q = 0$ (b) $2q^2 + p = 0$ (c) $q < 0$ (d) none of these

Q 117. The cubic equation whose roots are the AM, GM and HM of the roots of $x^2 - 2px + q^2 = 0$ is

(a) $(x - p)(x - q)(x - p - q) = 0$ (b) $(x - p)(x - |q|)(px - q^2) = 0$

(c) $x^3 - \left(p + |q| + \frac{q^2}{p}\right)x^2 + \left(p|q| + q^2 + \frac{|q|^3}{p}\right)x - |q|^3 = 0$ (d) none of these

Q 118. If $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, $a \neq b$, have a common root α then

(a) $a + b = 1$ (b) $\alpha + 1 = 0$ (c) $\alpha = 1$ (d) $a + b + 1 = 0$

Q 119. The line $y + 14 = 0$ cuts the curve whose equation is $x(x^2 + x + 1) + y = 0$ at

(a) three real points (b) one real point (c) at least one real point (d) no real point

Q 120. If a, b, c are in GP, where a, c are positive, then the equation $ax^2 + bc + c = 0$ has

(a) real roots (b) imaginary roots

(c) ratio of roots = 1 : w where w is a nonreal cube root of unity

(d) ratio of roots = b : ac

Q 121. If α, β are the roots of the equation $x^2 + x + 3 = 0$ then the equation $3x^2 + 5x + 3 = 0$ has a root

(a) $\frac{\alpha}{\beta}$ (b) $\frac{\beta}{\alpha}$ (c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (d) none of these

Q 122. If α, β are the roots of $x^2 - 2ax + b^2 = 0$ and γ, δ are the roots of $x^2 - 2bx + a^2 = 0$ then

(a) AM of α, β = GM of γ, δ (b) GM of α, β = AM of γ, δ

(c) $\alpha, \beta, \gamma, \delta$ are in AP (d) $\alpha, \beta, \gamma, \delta$ are in GP

Q 123. If the roots of the equation $ax^2 - 4x + a^2 = 0$ are imaginary and the sum of the roots is equal to their product then a is

(a) -2 (b) 4 (c) 2 (d) none of these

Q 124. If x, y, z are three consecutive terms of a GP, where $x > 0$ and the common ratio is r, then the inequality $z + 3x > 4y$ holds for

(a) $r \in (-\infty, 1)$ (b) $r = \frac{24}{5}$ (c) $r \in (3, +\infty)$ (d) $r = \frac{1}{2}$

Q 125. The equation $||x - 1| + a| = 4$ can have real solutions for x if a belongs to the interval

(a) $(-\infty, 4]$ (b) $(-\infty, -4]$ (c) $(4, +\infty)$ (d) $[-4, 4]$

Q 126. The equation $|x + 1|\ |x - 1| = a^2 - 2a - 3$ can have real solutions for x if a belongs to

(a) $(-\infty, -1] \cup [3, +\infty)$ (b) $[1 - \sqrt{5}, 1 + \sqrt{5}]$

(c) $[1-\sqrt{5}, -1] \cup [3, 1+\sqrt{5}]$ (d) none of these

Q 127. The common roots of the equations $x^3 + 2x^2 + 2x + 1 = 0$ and $1 + x^{130} + x^{1988} = 0$ are (where ω is a nonreal cube root of unity)

(a) ω (b) ω^2 (c) -1 (d) $\omega - \omega^2$

Q 128. If α is a root of the equation $2x(2x + 1) = 1$ then the other root is

(a) $3\alpha^3 - 4\alpha$ (b) $-2\alpha(\alpha + 1)$ (c) $4\alpha^3 - 3\alpha$ (d) none of these

Q 129. For the equation $2x^2 + 6\sqrt{2}\,x + 1 = 0$

(a) roots are rational

(b) if one root is $p+\sqrt{q}$ then the other is $-p+\sqrt{q}$

(c) roots are irrational

(d) if one root is $P+\sqrt{q}$ then the other is $p-\sqrt{q}$

Q 130. If α, β are the real roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$ then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always

(a) two real roots

(b) two negative roots

(c) two positive roots

(d) one positive root and one negative root

Q 131. The equation $x^{3/4(\log_2 x)^2 + \log_2 x - 5/4} = \sqrt{2}$ has

(a) at least one negative solution

(b) exactly ne irrational solution

(c) exactly three real solutions

(d) two nonreal complex roots

Q 132. If a, b, c are rational and no two of them are equal then the equations

$$(b - c)x^2 + (c - a)x + a - b = 0$$

and $$a(b - c)x^2 + b(c - a)x + c(a - b) = 0$$

(a) have rational roots

(b) will be such that at least one has rational roots

(c) have exactly one root common

(d) have at least one root common

Q 133. The equations $x^2 + b^2 = 1 - 2bx$ and $x^2 + a^2 = 1 - 2ax$ have one and only one root common. Then

(a) $a - b = 2$ (b) $a - b + 2 = 0$ (c) $|a - b| = 2$ (d) none of these

Q 134. If $px^2 + qx + r = 0$ has no real roots and p, q, r are real such that $p + r > 0$ then

(a) $p - q + r < 0$ (b) $p - q + r > 0$ (c) $p + r = 0$ (d) all of these

Q 135. Let p and q be roots of the equation $x^2 - 2x + A = 0$, and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmetic progression then

(a) A = -83, B = -3 (b) A = -3, B = 77 (c) q = 3, r = 7 (d) p + q + r + s = 20

Q 136. The quadratic equation $x^2 - 2x - \lambda = 0, \lambda \neq 0$

(a) cannot have a real root if $\lambda < -1$

(b) can have a rational root if λ is a perfect square

(c) cannot have an integral root if $n^2 - 1 < \lambda < n^2 + 2n$ where n = 0, 1, 2, 3, …..

(d) none of these

Q 137. A quadratic equation whose roots are $\left(\frac{\gamma}{\alpha}\right)^2$ and $\left(\frac{\beta}{\alpha}\right)^2$, where α, β, γ are the roots of $x^3 + 27 = 0$, is

(a) $x^2 - x + 1 = 0$ (b) $x^2 + 3x + 9 = 0$ (c) $x^2 + x + 1 = 0$ (d) $x^2 - 3x + 9 = 0$

Q 138. The graph of the curve $x^2 = 3x - y - 2$ is

(a) between the lines x = 1 and $x = \frac{3}{2}$ (b) between the lines x = 1 and x = 2

(c) strictly below the line 4y = 1 (d) none of these

Q 139. $a(x^2 - y^2) + \lambda\{x(y + 1) + 1\}$ can be resolved into linear rational factors. Then

(a) $\lambda = 1$ (b) $\lambda = \frac{4a^2}{a-1}, a \neq 1$ (c) $\lambda = 0, a = 1$ (d) none of these

Q 140. $x^2 - 4$ is a factor of $f(x) = (a_1x^2 + b_1x + c_1).(a_2x^2 + b_2x + c_2)$ if

(a) $b_1 = 0, c_1 + 4a_1 = 0$ (b) $b_2 = 0, c_2 + 4a_2 = 0$

(c) $4a_1 + 2b_1 + c_1 = 0, 4a_2 + c_2 = 2b_2$ (d) $4a_1 + c_1 = 2b_1, 4a_2 + 2b_2 + c_2 = 0$

Q 141. $ax^2 + by^2 + cz^2 + 2ayz + 2bzx + 2cxy$ can be resolved into liner factors if a, b, c are such that

(a) a = b = c (b) ab + bc + ca = 1 (c) a + b + c = 0 (d) none of these

Q 142. If a, b are the real roots of $x^2 + px + 1 = 0$ and c, d are the real roots of $x^2 + qx + 1 = 0$ then $(a - c)(b - c)(a + b)(b + d)$ is divisible by

(a) a + b + c + d (b) a + b – c – d (c) a – b + c – d (d) a – b – c – d

Q 143. If $x \in [2, 4]$ then for the expression $x^2 - 6x + 5$

(a) the least value = -4 (b) the greatest value = 4

(c) the least value = 3 (d) the greatest value = -4

Q 144. If 0 < a < 5, 0 < b < 5 and $\frac{x^2+5}{2} = x - 2\cos(a+bx)$ is satisfied for at least one real x then the greatest value of a + b is

(a) π (b) $\frac{\pi}{2}$ (c) 3π (d) 4π

Q 145. Let $f(x) = x^2(x + 2) + x + 3$. Then

(a) f(-3 –k) < 0 and f(-2 + k) > 0 for all k > 0 (b) f(-3 – k) > 0 and f(-2 + k) < 0 for all k > 0

(c) f(x) = 0 has a root α such that $[\alpha] + 3 = 0$, where $[\alpha]$ is the greatest integer less than or equal to α

(d) f(x) = 0 has exactly one root α such that $(\alpha) + 2 = 0$, where (α) is the smallest integer greater than or equal to α

Answers

1c	2a	3c	4a	5d	6b	7c	8a	9b	10a
11c	12b	13c	14a	15b	16b	17b	18a	19a	20b
21c	22c	23b	24d	25c	26b	27a	28d	29b	30d
31b	32b	33c	34a	35b	36a	37c	38c	39c	40b
41a	42b	43c	44a	45c	46a	47b	48a	49b	50b
51a	52c	53c	54a	55a	56a	57d	58b	59c	60b
61b	62b	63c	64c	65b	66b	67b	68b	69d	70c
71b	72a	73b	74c	75b	76c	77b	78b	79a	80c
81c	82b	83a	84d	85d	86b	87c	88b	89a	90c
91a	92b	93a	94c	95a	96d	97c	98d	99d	100b
101c	102b	103a	104a	105c	106b	107a	108c	109a	110b
111c	112b	113ac	114ac	115bc	116ac	117bc	118cd	119b	120bc
121ab	122ab	123c	124abcd	125ab	126ac	127ab	128bc	129bc	130ad
131bc	132ac	133abc	134b	135bcd	136ac	137c	138c	139bc	140abcd
141ac	142ab	143ad	144c	145acd					

Complex Numbers

Choose the most appropriate option (a, b, c or d).

Q 1. If a < 0, b > 0 then $\sqrt{a}.\sqrt{b}$ is equal to

(a) $-\sqrt{|a|.b}$ (b) $\sqrt{|a|.b}.i$ (c) $\sqrt{|a|b}$ (d) none of these

Q 2. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, is

(a) i (b) i – 1 (c) –i (d) 0

Q 3. If n_1, n_2 are positive integers then

$$(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$$

is a real number if and only if

(a) $n_1 = n_2 + 1$ (b) $n_1 + 1 = n_2$ (c) $n_1 = n_2$ (d) n_1, n_2 are any two positive integers

Q 4. The complex number $\frac{2^n}{(1+i)^{2n}} + \frac{(1+i)^{2n}}{2^n}, \frac{2^n}{(1+i)^{2n}} + \frac{(1+i)^{2n}}{2^n}, n \in \mathbb{Z}$

(a) 0 (b) 2 (c) $\{1 + (-1)^n\} . i^n$ (d) none of these

Q 5. The smallest positive integral value of n for which $\left(\frac{1-i}{1+i}\right)^n$ is purely imaginary with positive imaginary part, is

(a) 1 (b) 3 (c) 5 (d) none of these

Q 6. If $(a + ib)^5 + \alpha + i\beta$ then $(b + ia)^5$ is equal to

(a) $\beta + i\alpha$ (b) $\alpha - i\beta$ (c) $\beta - i\alpha$ (d) $-\alpha - i\beta$

Q 7. If $i = \sqrt{-1}$, the number of values of $i^n + i^{-n}$ for different $n \in \mathbb{Z}$ is

(a) 3 (b) 2 (c) 4 (d) 1

Q 8. Im(z) is equal to

(a) $\frac{1}{2}(z+\bar{z})i$ (b) $\frac{1}{2}(z-\bar{z})$ (c) $\frac{1}{2}(\bar{z}-z)i$ (d) none of these

Q 9. The value of $(1 + i)^3 + (1 – i)^6$ is

(a) i (b) 2(-1 + 5i) (c) 1 – 5i (d) none of these

Q 10. Taking the value of a square root with positive real part only, the value of $\sqrt{-3-4i}+\sqrt{3+4i}$ is

(a) 1 + i (b) 1 – 3i (c) 1 + 3i (d) none of these

Q 11. $\sin^{-1}\left\{\frac{1}{i}(z-1)\right\}$, where z is nonreal, can be the angle of a triangle if

(a) Re(z) = 1, Im(z) = 2 (b) Re(z) = 1, $-1 \le \text{Im}(z) \le 1$ (c) Re(z) + Im(z) = 0 (d) none of these

Q 12. If n is an odd integer, $i=\sqrt{-1}$ then $(1 + i)^{6n} + (1 – i)^{6n}$ is equal to

(a) 0 (b) 2 (c) -2 (d) none of these

Q 13. If $z_1 = 9y^2 – 4 – 10ix$, $z_2 = 8y^2 – 20i$, where $z_1 = \bar{z}_2$, then z = x + iy is equal to

(a) -2 + 2i (b) $-2 \pm 2i$ (c) $-2 \pm i$ (d) none of these

Q 14. The complex numbers sin x – icos 2x and cos x – isin 2x are conjugate to each other for

(a) $x = n\pi$ (b) x = 0 (c) $x = (2n + 1)\frac{\pi}{2}$ (d) no value of x

Q 15. If $z = 1 + i\tan\alpha$, where $\pi < \alpha < \frac{3\pi}{2}$, then $|z|$ is equal to

(a) $\sec\alpha$ (b) $-\sec\alpha$ (c) $\text{cosec}\,\alpha$ (d) none of these

Q 16. If z is a complex number satisfying the reaction $|z + 1| = z + 2(1 + i)$ then z is

(a) $\frac{1}{2}(1+4i)$ (b) $\frac{1}{2}(3+4i)$ (c) $\frac{1}{2}(1-4i)$ (d) $\frac{1}{2}(3-4i)$

Q 17. If $(1 + i)z = (1 + i)\bar{z}$ then z is

(a) $t(1 – i), t \in R$ (b) $t(1 + i), t \in R$ (c) $\frac{t}{1+i}, t \in R^+$ (d) none of these

Q 18. If z_1, z_2 are two nonzero complex numbers such that

$|z_1 + z_2| = |z_1| + |z_2|$ then amp $\frac{z_1}{z_2}$ is equal to

(a) π (b) $-\pi$ (c) 0 (d) none of these

Q 19. The complex number z is purely imaginary if

(a) $z\bar{z}$ is real (b) $z = \bar{z}$ (c) $z + \bar{z} = 0$ (d) none of these

Q 20. If z = x + iy such that |z + 1| = |z – 1| and amp $\frac{z-1}{z+1}=\frac{\pi}{4}$ then

(a) $x=\sqrt{2}+1, y=0$ (b) $x=0, y=\sqrt{2}+1$ (c) $x=0, y=\sqrt{2}-1$ (d) $x=\sqrt{2}-1, y=0$

Q 21. Let $z=\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta}, \frac{\pi}{4}<\theta<\frac{\pi}{2}$. Then arg z is

(a) 2θ (b) $2\theta - \pi$ (c) $\pi + 2\theta$ (d) none of these

Q 22. If $z=\frac{\sqrt{3}+i}{\sqrt{3}-i}$ then the fundamental amplitude of z is

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) none of these

Q 23. If $\frac{1+2i}{2+i}$ = r(cos θ + isin θ) then

(a) r = 1, $\theta = \tan^{-1}\frac{3}{4}$ (b) $r=\sqrt{5}, \theta=\tan^{-1}\frac{4}{3}$ (c) $r=1, \theta=\tan^{-1}\frac{4}{3}$ (d) none of these

Q 24. If z = x + iy satisfies amp (z – 1) = amp (z + 3i) then the value of (x – 1) : y is equal to

(a) 2 : 1 (b) 1 : 3 (c) -1 : 3 (d) none of these

Q 25. Let z be a complex number of constant modulus such that z^2 is purely imaginary then the number of possible values of z is

(a) 2 (b) 1 (c) 4 (d) infinite

Q 26. If ω is an imaginary cube root of unity then $(1 + \omega - \omega^2)^7$ equals

(a) 128ω (b) -128ω (c) $128\omega^2$ (d) $-128\omega^2$

Q 27. If ω is a nonreal cube root of unity then the expression

$(1 - \omega)(1 - \omega^2)(1 + \omega^4)(1 + \omega^8)$ is equal to

(a) 0 (b) 3 (c) 1 (d) 2

Q 28. If $3^{49}(x + iy) = \left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^{100}$ and x = ky then k is

(a) $-\frac{1}{3}$ (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $-\frac{1}{\sqrt{3}}$

Q 29. $x^{3m} + x^{3n-1} + x^{3r-2}$, where m, n, r, ∈ N, is divisible by

(a) $x^2 - x + 1$ (b) $x^2 + x + 1$ (c) $x^2 + x - 1$ (d) $x^2 - x - 1$

Q 30. If $x^2 - x + 1 = 0$ then the value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$ is

(a) 8 (b) 10 (c) 12 (d) none of these

Q 31. If $1 + x^2 = \sqrt{3}x$ then $\sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)^2$ is equal to

(a) 48 (b) -48 (c) $\pm 48(\omega - \omega^2)$ (d) none of these

Q 32. The smallest positive integral value of n for which $(1 + \sqrt{3}i)^{n/2}$ is real is

(a) 3 (b) 6 (c) 12 (d) 0

Q 33. If $i = \sqrt{-1}$, ω = nonreal cube root of unity then

$$\frac{(1+i)^{2n} - (1-i)^{2n}}{(1+\omega^4 - \omega^2)(1-\omega^4+\omega^2)}$$ is equal to

(a) 0 if n is even (b) o for all $n \in Z$

(c) 2^{n-1}. i for all $n \in N$ (d) none of these

Q 34. If $z^2 - z + 1 = 0$ then $z^n - z^{-n}$, where n is a multiple of 3, is

(a) $2(-1)^n$ (b) 0 (c) $(-1)^{n+1}$ (d) none of these

Q 35. If ω is a nonreal cube root of unity then

$$\frac{1+2\omega+3\omega^2}{2+3\omega+\omega^2} + \frac{2+3\omega+\omega^2}{3+\omega+2\omega^2}$$ is equal to

(a) -1 (b) 2ω (c) 0 (d) -2ω

Q 36. If $(x-1)^4 - 16 = 0$ then the sum of nonreal complex values of x is

(a) 2 (b) 0 (c) 4 (d) none of these

Q 37. If $z_r = \cos\frac{2r\pi}{5} + i\sin\frac{2r\pi}{5}$, r = 0, 1, 2, 3, 4,….. then $z_1z_2z_3z_4z_5$ is equal to

(a) -1 (b) 0 (c) 1 (d) none of these

Q 38. If $e^{i\theta} = \cos\theta + i\sin\theta$ then for the ΔABC, $e^{iA} . e^{iB} . e^{iC}$ is

(a) $-i$ (b) 1 (c) -1 (d) none of these

Q 39. If $(\sqrt{3}+i)^n = (\sqrt{3}-i)^n$, $n \in N$ then the least value of n is

(a) 3 (b) 4 (c) 6 (d) none of these

Q 40. If the fourth roots of unity are z_1, z_2, z_3, z_4 then $z_1^2+z_2^2+z_3^2+z_4^2$ is equal to

(a) 1 (b) 0 (c) i (d) none of these

Q 41. If $x^3 - 1 = 0$ has the nonreal complex roots α, β then the value of $(1 + 2\alpha + \beta)^3 - (3 + 3\alpha + 5\beta)^3$ is

(a) -7 (b) 6 (c) -5 (d) 0

Q 42. If $i=\sqrt{-1}$ then $4+5\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{334}-3\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{365}$ is equal to

(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$ (c) $4\sqrt{3}i$ (d) $-i\sqrt{3}$

Q 43. If $(\sqrt{3}-i)^n = 2^n$, $n \in \mathbb{Z}$, the set of integers, then n is a multiple of

(a) 6 (b) 10 (c) 9 (d) 12

Q 44. If $z(2 - i2\sqrt{3})^2 = i(\sqrt{3}+i)^4$ the amplitude of z is

(a) $\frac{5\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{7\pi}{6}$

Q 45. If z is a nonreal root of $\sqrt[7]{-1}$ then $z^{86} + z^{175} + z^{289}$ is equal to

(a) 0 (b) -1 (c) 3 (d) 1

Q 46. If α is nonreal and $\alpha = \sqrt[5]{1}$ then the value of $2^{|1+\alpha+\alpha^2+\alpha^{-2}-\alpha^{-1}|}$ is equal to

(a) 4 (b) 2 (c) 1 (d) none of these

Q 47. The value of amp $(i\omega)$ + amp $(i\omega^2)$, where $i=\sqrt{-1}$ and $\omega=\sqrt[3]{1}$ = nonreal, is

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) none of these

Q 48. If α, β be two complex numbers then $|\alpha^2| + |\beta|^2$ is equal to

(a) $\frac{1}{2}(|\alpha+\beta|^2-|\alpha-\beta|^2)$ (b) $\frac{1}{2}(|\alpha+\beta|^2+|\alpha-\beta|^2)$

(c) $|\alpha+\beta|^2+|\alpha-\beta|^2$ (d) none of these

Q 49. The set of values of $a \in R$ for which $x^2 + i(a-1)x + 5 = 0$ will have a pair conjugate complex roots is

(a) R (b) {1} (c) $\{a \mid a^2 - 2a + 21 > 0\}$ (d) none of these

Q 50. Nonreal complex numbers z satisfying the equation $z^3 + 2z^2 + 3z + 2 = 0$ are

(a) $\frac{-1 \pm \sqrt{-7}}{2}$ (b) $\frac{1+\sqrt{7}i}{2}, \frac{1-\sqrt{7}i}{2}$ (c) $-i, \frac{-1+\sqrt{7}i}{2}, \frac{-1-\sqrt{7}i}{2}$ (d) none of these

Q 51. For a complex number z, the minimum value of $|z| + |z-2|$ is

(a) 1 (b) 2 (c) 3 (d) none of these

Q 52. If $|z| = 1$ then $\frac{1+z}{1+\bar{z}}$ is equal to

(a) z (b) $\bar{z}$ (c) $z + \bar{z}$ (d) none of these

Q 53. If α is a nonreal cube root of unity then $|\alpha^n|$, $n \in Z$, is equal to

(a) 1 (b) 3 (c) 0 (d) none of these

Q 54. If z be a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$ then $|z|$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) 1 (d) none of these

Q 55. Let $z_1 = a + ib$, $z_2 = p + iq$ be two unimodular complex numbers such that $\mathrm{Im}(z_1\bar{z}_2) = 1$. If $\omega_1 = a + ip$, $\omega_2 = b + iq$ then

(a) $\mathrm{Re}(\omega_1\omega_2) = 1$ (b) $\mathrm{Im}(\omega_1\omega_2) = 1$ (c) $\mathrm{Re}(\omega_1\omega_2) = 0$ (d) $\mathrm{Im}(\omega_1\bar{\omega}_2) = 1$

Q 56. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$

(a) is less than 6 (b) is more than 3 (c) is less than 12 (d) lies between 6 and 12

Q 57. If $|z - i| \le 2$ and $z_0 = 5 + 3i$ then the maximum value of $|iz + z_0|$ is

(a) $2+\sqrt{31}$ (b) 7 (c) $\sqrt{31}-2$ (d) none of these

Q 58. If $|z| = \max\{|z-1|, |z+1|\}$ then

(a) $|z_1 + \bar{z}| = \frac{1}{2}$ (b) $z_1 + \bar{z} = 1$ (c) $|z_1 + \bar{z}| = 1$ (d) none of these

Q 59. $|z - 4| < |z - 2|$ represents the region given by

(a) $\mathrm{Re}(z) > 0$ (b) $\mathrm{Re}(z) < 0$ (c) $\mathrm{Re}(z) > 2$ (d) none of these

Q 60. If $\log_{1/2}\frac{|z|^2+2|z|+4}{2|z|^2+1}<0$ then the region traced by z is

(a) $|z| < 3$ (b) $1 < |z| < 3$ (c) $|z| > 1$ (d) $|z| < 2$

Q 61. $\left|\frac{z-1}{z+1}\right|=1$ represents

(a) a circle (b) an ellipse (c) a straight line (d) none of these

Q 62. If $2z_1 - 3z_2 + z_3 = 0$ then z_1, z_2, z_3 are represented by

(a) three vertices of a triangle (b) three collinear points

(c) three vertices of a rhombus (d) none of these

Q 63. If A, B, C are three points in the Argand plane representing the complex numbers z_1, z_2, z_3 such that $z_1=\frac{\lambda z_2+z_3}{\lambda+1}$, where $\lambda \in R$, then the distance of A from the line BC is

(a) λ (b) $\frac{\lambda}{\lambda+1}$ (c) 1 (d) 0

Q 64. The roots of the equation $1 + z + z^3 + z^4 = 0$ are represented by the vertices of

(a) a square (b) an equilateral triangle (c) a rhombus (d) none of these

Q 65. If $\mathrm{Re}\left(\frac{z+4}{2z-i}\right)=\frac{1}{2}$ then z is represented by a point lying on

(a) a circle (b) an ellipse (c) a straight line (d) none of these

Q 66. The angle that the vector representing the complex number $\frac{1}{(\sqrt{3}-i)^{25}}$ makes with the positive direction of the real axis is

(a) $\frac{2\pi}{3}$ (b) $-\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

Q 67. If P, P' represent the complex number z_1 and its additive inverse respectively then the complex equation of the circle with PP' as a diameter is

(a) $\frac{z}{z_1}=\left(\frac{\bar{z}_1}{z}\right)$ (b) $z\bar{z}+z_1\bar{z}_1=0$ (c) $z\bar{z}_1+\bar{z}z_1=0$ (d) none of these

Q 68. If $|z_1| = |z_2| = |z_3| = |z_4|$ then the points representing z_1, z_2, z_3, z_4 are

(a) concyclic (b) vertices of a square

(c) vertices of a rhombus (d) none of these

Q 69. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + \sqrt{3}i$ and z_1, z_2, z_3 are in the clockwise sense then

(a) $z_1 = 1-\sqrt{3}i$, $z_3 = -2$ (b) $z_2 = 2$, $z_3 = 1 - \sqrt{3}i$ (c) $z_2= -1+\sqrt{3}i$, $z_3 = -2$ (d) none of these

Q 70. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle circumscribing the circle $|z| = 1$. If $z_1 = 1 + \sqrt{3}i$ and z_1, z_2, z_3 are in the anticlockwise sense then z_2 is

(a) $1-\sqrt{3}i$ (b) 2 (c) $\frac{1}{2}(1-\sqrt{3}i)$ (d) none of these

Q 71. If amp $\frac{z-1}{z+1} = \frac{\pi}{3}$ then z represents a point on

(a) a straight line (b) a circle (c) a pair of lines (d) none of these

Q 72. If the roots of $z^3 + iz^2 + 2i = 0$ represent the vertices of a ΔABC in the Argand plane then the area of the triangle is

(a) $\frac{3\sqrt{7}}{2}$ (b) $\frac{3\sqrt{7}}{4}$ (c) 2 (d) none of these

Q 73. The equation $z\bar{z} + (4 - 3i)z + (4 + 3i)\bar{z} + 5 = 0$ represents a circle whose radius is

(a) 5 (b) $2\sqrt{5}$ (c) $\frac{5}{2}$ (d) none of these

Q 74. If z is a complex number such that $\left|\frac{z-3i}{z+3i}\right| = 1$ then z lies on

(a) the real axis (b) the line $Im(z) = 3$ (c) a circle (d) none of these

Q 75. Let z_1 and z_2 be two nonreal complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is

(a) 4 (b) 3 (c) 2 (d) $\sqrt{2}$

Q 76. Let $\lambda \in R$. If the origin and the nonreal roots of $2z^2 + 2z + \lambda = 0$ form the three vertices of an equilateral triangle in the Argand plane then λ is

(a) 1 (b) $\frac{2}{3}$ (c) 2 (d) 1

Q 77. The equation $|z - i| + |z + i| = k$, $k > 0$, can represent an ellipse if k is

(a) 1 (b) 2 (c) 4 (d) none of these

Q 78. The equation $|z + i| - |z - i| = k$ represents a hyperbola if

(a) $-2 < k < 2$ (b) $k > 2$ (c) $0 < k < 2$ (d) none of these

Q 79. Let OP.OQ = 1 and let O, P, Q be three collinear points. If O and Q represent the complex numbers 0 and z then P represents

(a) $\frac{1}{z}$ (b) $\bar{z}$ (c) $\frac{1}{\bar{z}}$ (d) none of these

Q 80. Let $z = 1 - t + i\sqrt{t^2 + t + 2}$, where t is a real parameter. Then locus of z in the Argand plane is

(a) a hyperbola (b) an ellipse (c) a straight line (d) none of these

Q 81. The area of the triangle whose vertices are i, α, β, where $i = \sqrt{-1}$ and α, β are the nonreal cube roots of unity, is

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{4}$ (c) 0 (d) $\frac{\sqrt{3}}{4}$

Choose the correct options. One or more options may be correct.

Q 82. The nonzero real value of x for which $\frac{(1+ix)(1+2ix)}{1-ix}$ is purely real is

(a) $\sqrt{2}$ (b) 1 (c) $-\sqrt{2}$ (d) none of these

Q 83. If $z_1 = \frac{1}{a+i}, a \neq 0$ and $z_2 = \frac{1}{1+bi}, b \neq 0$ such that $z_1 = \bar{z}_2$ then

(a) a = 1, b = 1 (b) a = -1, b = 1 (c) a = 1, b = -1 (d) none of these

Q 84. If z_1, z_2, z_3, z_4 are roots of the equation

$$a_0z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$$

where a_0, a_1, a_2, a_3 and a_4 are real, then

(a) $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$ are also roots of the equation (b) z_1 is equal to at least one of $\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4$

(c) $-\bar{z}_1, -\bar{z}_2, -\bar{z}_3, -\bar{z}_4$ are also roots of the equation (d) none of these

Q 85. If α is a complex constant such that $\alpha z^2 + z + \bar{\alpha} = 0$ has a real root then

(a) $\alpha + \bar{\alpha} = 1$ (b) $\alpha + \bar{\alpha} = 0$ (c) $\alpha + \bar{\alpha} = -1$

(d) the absolute value of the real roots is 1

Q 86. If $\text{amp}(z_1z_2) = 0$ and $|z_1| = |z_2| = 1$ then

(a) $z_1 + z_2 = 0$ (b) $z_1z_2 = 1$ (c) $z_1 = \bar{z}_2$ (d) none of these

Q 87. If z is a nonzero complex number then $\left|\frac{|\bar{z}|^2}{z\bar{z}}\right|$ is equal to

(a) $\left|\frac{\bar{z}}{z}\right|$ (b) 1 (c) $|\bar{z}|$ (d) none of these

Q 88. If ω is a nonreal cube root of unity then the value of

$1.(2 - \omega)(2 - \omega^2) + 2. (3 - \omega)(3 - \omega^2) + ... + (n - 1)(n - \omega)(n - \omega^2)$ is

(a) real (b) $\frac{n^2(n-1)^2}{4} - n + 1$ (c) $\left\{\frac{n(n+1)^2}{2}\right\} - n$ (d) not real

Q 89. If z is a complex number satisfying $z + z^{-1} = 1$ then $z^n + z^{-n}$, $n \in N$, has the value

(a) $2(-1)^n$ when n is a multiple of 3 (b) $(-1)^{n-1}$ when n is not a multiple of 3

(c) $(-1)^{n+1}$ when n is a multiple of 3 (d) 0 when n is not a multiple of 3

Q 90. The value of $\alpha^{-n} + \alpha^{-2n}$, $n \in N$ and α is a nonreal cube root of unity, is

(a) 3 if n is a multiple of 3 (b) -1 if n is not a multiple of 3

(c) 2 if n is a multiple of 3 (d) none of these

Q 91. The value of $\alpha^{4n-1} + \alpha^{4n-2} + \alpha^{4n-3}$, $n \in N$ and α is a nonreal fourth root of unity, is

(a) 0 (b) -1 (c) 3 (d) none of these

Q 92. Let x be a nonreal complex number satisfying $(x - 1)^3 + 8 = 0$ then x is

(a) $1 + 2\omega$ (b) $1 - 2\omega$ (c) $1 - 2\omega^2$ (d) none of these

Q 93. If $z = \frac{1+3i}{1+i}$ then

(a) Re(z) = 2Im(z) (b) Re(z) + 2Im(z) = 0 (c) $|z| = \sqrt{5}$ (d) amp z = $\tan^{-1}2$

Q 94. If z is different from $\pm$ i and $|z| = 1$ then $\frac{z+i}{z-i}$ is

(a) purely real (b) nonreal, whose real and imaginary parts are equal

(c) purely imaginary (d) none of these

Q 95. If z_1, z_2 are two compelx numbers then

(a) $|z_1 + z_2| \le |z_1| + |z_1|$ (b) $|z_1 - z_2| \ge |z_1| - |z_2|$

(c) $|z_1 + z_2| \ge |z_1 . z_2|$ (d) $|z_1 - z_2| \le |z_1 + z_2|$

Q 96. Let z_1, z_2 be two complex numbers represented by points on the circle $|z| = 1$ and $|z| = 2$ respectively then

(a) max $|2z_1 + z_2| = 4$ (b) min $|z_1 - z_2| = 1$

(c) $\left|z_2 + \frac{1}{z_1}\right| \le 3$ (d) none of these

Q 97. ABCD is a square, vertices being taken in the anticlockwise sense. If A represents the complex number z and the intersection of the diagonals is the origin then

(a) B represents the complex number iz (b) D represents the complex number $i\bar{z}$

(c) B represents the complex number $i\bar{z}$ (d) D represents the complex number $-i\bar{z}$

Q 98. If $z(\overline{z+\alpha}) + \bar{z}(z+\alpha) = 0$, where α is a complex constant, then z is represented by a point on

(a) a straight line (b) a circle (c) a parabola (d) none of these

Q 99. If z_1, z_2, z_3, z_4 are the four complex numbers represented by the vertices of a quadrilateral taken in order such that $z_1 - z_4 = z_2 - z_3$ and amp $\frac{z_4 - z_1}{z_2 - z_1} = \frac{\pi}{2}$ then the quadrilateral is a

(a) rhombus (b) square (c) rectangle (d) a cyclic quadrilateral

Q 100. If z_0, z_1 represent point P, Q on the locus $|z - 1| = 1$ and the line segment PQ subtends and angle $\pi/2$ at the point z = 1 then z_1 is equal to

(a) $1 + i(z_0 - 1)$ **(b)** $\frac{i}{z_0 - 1}$ **(c)** $1 - i(z_0 - 1)$ **(d)** $i(z_0 - 1)$

Q 101. If $|z_1| = |z_2| = |z_3| = 1$ and z_1, z_2, z_3 are represented by the vertices of an equilateral triangle then

(a) $z_1 + z_2 + z_3 = 0$ (b) $z_1z_2z_3 = 1$ (c) $z_1z_2 = z_2z_3 + z_3z_1 = 0$ (d) none of these

Q 102. Let A, B, C be three collinear points which are such that AB.AC = 1 and the points are represented in the Argand plane by the line complex numbers 0, z_1, z_2 respectively. Then

(a) $z_1z_2 = 1$ (b) $z_1\bar{z}_2 = 1$ (c) $|z_1||z_2| = 1$ (d) none of these

Q 103. If z_1, z_2, z_3, z_4 are represented by the vertices of a rhombus taken in the anticlockwise order then

(a) $z_1 - z_2 + z_3 - z_4 = 0$ (b) $z_1 + z_2 = z_3 + z_4$ (c) $\text{amp}\frac{z_2 - z_4}{z_1 - z_3} = \frac{\pi}{2}$ (d) $\text{amp}\frac{z_1 - z_2}{z_3 - z_4} = \frac{\pi}{2}$

Q 104. If $\text{amp}\frac{z-2}{2z+3i} = 0$ and $z_0 = 3 + 4i$ then

(a) $z_0\bar{z} + \bar{z}_0 z = 12$ (b) $z_0 z + \bar{z}_0\bar{z} = 12$ (c) $z_0\bar{z} + \bar{z}_0 z = 0$ (d) none of these

Q 105. If $z_1 \neq z_2$ and $|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$ then

(a) at least one of $z_1.z_2$ is unimodular (b) both z_1, z_2 are unimodular

(c) $z_1.z_2$ is unimodular (d) none of these

Q 106. Let $z_1 = \frac{(\sqrt{3}+i)^2.(1-\sqrt{3}i)}{1+i}, z_2 = \frac{(1+\sqrt{3}i)^2.(\sqrt{3}-i)}{1-i}$. Then

(a) $|z_1| = |z_2|$ (b) amp z_1 + amp $z_2 = 0$ (c) $3|z_1| = |z_2|$ (d) 3amp z_1 + amp $z_2 = 0$

Q 107. If $|z_1 + z_2| = |z_1 - z_2|$ then

(a) $|\text{amp } z_1 - \text{amp } z_2| = \frac{\pi}{2}$ (b) $|\text{amp } z_1 - \text{amp } z_2| = \pi$

(c) $\frac{z_1}{z_2}$ is purely real (d) $\frac{z_1}{z_2}$ is purely imaginary

Q 108. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then

(a) $\frac{z_1}{z_2}$ is purely real (b) $\frac{z_1}{z_2}$ is purely imaginary (c) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (d) $\text{amp}\frac{z_1}{z_2} = \frac{\pi}{2}$

Answers

1b	2b	3d	4c	5b	6a	7a	8c	9b	10d
11b	12a	13b	14d	15b	16c	17a	18c	19c	20b
21a	22b	23a	24b	25c	26d	27b	28d	29b	30a
31b	32b	33a	34b	35b	36a	37c	38c	39c	40b
41a	42c	43d	44b	45b	46a	47c	48b	49b	50a
51b	52a	53a	54c	55d	56c	57b	58c	59d	60a
61c	62b	63d	64b	65c	66d	67a	68a	69a	70d
71b	72c	73b	74a	75b	76b	77c	78a	79c	80a
81d	82ac	83c	84ab	85acd	86bc	87ab	88ab	89ab	90bc
91b	92bc	93ac	94c	95ab	96abc	97ad	98b	99cd	100ac
101ab	102bc	103ac	104b	105c	106ad	107ad	108bcd		

Permutation and Combination

Choose the most appropriate option (a, b, c or d).

Q 1. If ${}^{n}C_{r-1} = 56$, ${}^{n}C_{r} = 28$ and ${}^{n}C_{r+1} = 8$ then r is equal to

(a) 8 (b) 6 (c) 5 (d) none of these

Q 2. The value of ${}^{20}C_{31} + \sum_{j=0}^{10} {}^{40+j}C_{10+j}$ is equal to

(a) ${}^{51}C_{20}$ (b) $2.{}^{50}C_{20}$ (c) $2.{}^{45}C_{15}$ (d) none of these

Q 3. In a group of boys, the number of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The number of boys in the group is

(a) 10 (b) 8 (c) 6 (d) none of these

Q 4. The value of $\sum_{r=1}^{10} r.{}^{r}P_{r}$ is

(a) ${}^{11}P_{11}$ (b) ${}^{11}P_{11} - 1$ (c) ${}^{11}P_{11} + 1$ (d) none of these

Q 5. From a group of persons the number of ways of selecting 5 persons is equal to that of 8 persons. The number of persons in the group is

(a) 13 (b) 40 (c) 18 (d) 21

Q 6. The number of distinct rational numbers x such that $0 < x < 1$ and $x = \frac{p}{q}$, where p, q $\in$ {1, 2, 3, 4, 5, 6}, is

(a) 15 (b) 13 (c) 12 (d) 11

Q 7. The total number of 9-digit numbers of different digits is

(a) 10(9!) (b) 8(9!) (c) 9(9!) (d) none of these

Q 8. The number of 6-digit numbers that can be made with the digits 0, 1, 2, 3, 4 and 5 so that even digits occupy odd places, is

(a) 24 (b) 36 (c) 48 (d) none of these

Q 9. The number of ways in which 6 men can be arranged in a row so that three particular men are consecutive, is

(a) ${}^{4}P_{4}$ (b) ${}^{4}P_{4} \times {}^{3}P_{3}$ (c) ${}^{3}P_{3} \times {}^{3}P_{3}$ (d) none of these

Q 10. Seven different lectures are to deliver lectures in seven periods of a class on a particular day. A, B and C are three of the lectures. The number of ways in which a routine for the day can be made such that A delivers his lecture before B, and B before C, is

(a) 420 (b) 120 (c) 210 (d) none of these

Q 11. The total number of 5-digit numbers of different digits in which the digit in the middle is the largest is

(a) $\sum_{n=4}^{9} {}^nP_4$ (b) 33(3!) (c) 30(3!) (d) none of these

Q 12. A 5-digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 600 (c) 240 (d) 3125

Q 13. Let A = {x | x is a prime number and x < 30}. The number of different rational numbers whose numerator and denominator belong to A is

(a) 90 (b) 180 (c) 91 (d) none of these

Q 14. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is

(a) $\frac{7!}{3!}$ (b) $6! \times \frac{7!}{3!}$ (c) 35 (d) none of these

Q 15. The total number of words that can be made by writing the letters of the word PARAMETER so that no vowel is between two consonants is

(a) 1440 (b) 1800 (c) 2160 (d) none of these

Q 16. The number of numbers of four different digits that can be formed from the digits of the number 12356 such that the numbers are divisible by 4, is

(a) 36 (b) 48 (c) 12 (d) 24

Q 17. Let S be the set of all functions from the set A to the set A. If n(A) = k then n(S) is

(a) k! (b) k^k (c) $2^k - 1$ (d) 2^k

Q 18. Let A be the set of 4-digit numbers $a_1a_2a_3a_4$ where $a_1 > a_2 > a_3 > a_4$ then n(A) is equal to

(a) 126 (b) 84 (c) 210 (d) none of these

Q 19. The number of numbers divisible by 3 that can be formed by four different even digits is

(a) 18 (b) 36 (c) 0 (d) none of these

Q 20. The number of 5-digit even number that can be made with the digit 0, 1, 2 and 3 is

(a) 384 (b) 192 (c) 768 (d) none of these

Q 21. The number of 4-digit numbers that can be made with the digit 1, 2, 3, 4 and 5 in which at least two digits are identical, is

(a) $4^5 - 5!$ (b) 505 (c) 600 (d) none of these

Q 22. The number of words that can be made by rearranging the letters of the word APURBA so that vowels and consonants alternate is

(a) 18 (b) 35 (c) 36 (d) none of these

Q 23. The number of words that can be made by writing down the letters of the word CALCULATE such that each word starts and ends with a constant, is

(a) $\frac{5(7!)}{2}$ (b) $\frac{3(7!)}{2}$ (c) $2(7!)$ (d) none of these

Q 24. The number of numbers of 9 different nonzero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is

(a) $2(4!)$ (b) $(4!)^2$ (c) $8!$ (d) none of these

Q 25. In the decimal system of numeration the number of 6-digit numbers in which the digit in any place is greater than the digit to the left of it is

(a) 210 (b) 84 (c) 126 (d) none of these

Q 26. The number of 5-digit numbers in which no two consecutive digits are identical is

(a) $9^2 \times 8^3$ (b)

Q 27. In the decimal system of numeration the number of 6-digit numbers in which the sum of the digits is divisible by 5 is

(a) 180000 (b) 540000 (c) 5×10^5 (d) none of these

Q 28. The sum of all the numbers of four different digits that can be made by using the digits 0, 1, 2 and 3 is

(a) 26664 (b) 39996 (c) 38664 (d) none of these

Q 29. A teacher takes 3 children from her class to the zoo at a time as often as she can, but she does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is

(a) 12 (b) 10 (c) 60 (d) none of these

Q 30. ABCD is a convex quadrilateral. 3, 4, 5 and 6 points are marked on the sides AB, CD and DA respectively. The number of triangles with vertices on different sides is

(a) 270 (b) 220 (c) 282 (d) none of these

Q 31. There are 10 points in a plane of which no three points are collinear and 4 points are concyclic. The number of different circles that can be drawn through at least 3 points of these points is

(a) 116 (b) 120 (c) 117 (d) none of these

Q 32. In a polygon the number of diagonals is 54. The number of sides of the polygon is

(a) 10 (b) 12 (c) 9 (d) none of these

Q 33. In a polygon no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70 then the number of diagonals of polygon is

(a) 20 (b) 28 (c) 8 (d) none of these

Q 34. n lines are drawn in a plane such that no two of them are parallel and no three of them are concurrent. The number of different points at which these lines will cut is

(a) $\sum_{k=1}^{n-1} k$ (b) n(n -1) (c) n^2 (d) none of these

Q 35. The number of triangles that can be formed with 10 points as vertices, n of them being collinear, is 110. Then n is

(a) 3 (b) 4 (c) 5 (d) 6

Q 36. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices at these points is

(a) $3p^2(p - 1) + 1$ (b) $3p^2(p - 1)$ (c) $p^2(4p - 3)$ (d) none of these

Q 37. Two teams are to play a series of 5 matches between them. A match ends in a win or loss or draw for a team. A number of people forecast the result of each match and no two people make the same forecast for the series of matches. The smallest group of people in which one person forecasts correctly for all the matches will contain n people, where n is

(a) 81 (b) 243 (c) 486 (d) none of these

Q 38. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. The number of selections of at most 6 balls containing balls of all the colours is

(a) 42(4!) (b) $2^6 \times 4!$ (c) $(2^6 - 1)(4!)$ (d) none of these

Q 39. In a room there are 12 bulbs of the same wattage, each having a separate switch. The number of ways to light the room with different amounts of illumination is

(a) $12^2 - 1$ (b) 2^{12} (c) $2^{12} - 1$ (d) none of these

Q 40. In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is

(a) 255 (b) 256 (c) 193 (d) 319

Q 41. The number of 5-digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different, is

(a) 30 (b) 31 (c) 32 (d) none of these

Q 42. In a club electron the number contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways in which a voter can be 62 then the number of candidates is

(a) 7 (b) 5 (c) 6 (d) none of these

Q 43. The total number of selections of at most n things from (2n + 1) different things is 63. Then the value of n is

(a) 3 (b) 2 (c) 4 (d) none of these

Q 44. Let $1 \leq m < n \leq p$. The number of subsets of the set A = {1, 2, 3,….., p} having m, n as the least and the greatest elements respectively, is

(a) $2^{n-m-1} - 1$ (b) 2^{n-m-1} (c) 2^{n-m} (d) none of these

Q 45. The number of ways in which n different prizes can be distributed among m(<n) persons if each is entitled to receive at most n – 1 prizes, is

(a) $n^m - n$ (b) m^n (c) mn (d) none of these

Q 46. The number of possible outcomes in a throw of n ordinary dice in which at least one of the dice shows an odd number is

(a) $6^n - 1$ (b) $3^n - 1$ (c) $6^n - 3^n$ (d) none of these

Q 47. The number of different 6-digit numbers that can be formed using the three digits 0, 1 and 2 is

(a) 3^6 (b) 2×3^5 (c) 3^5 (d) none of these

Q 48. The number of different matrices that can be formed with elements 0, 1, 2 or 3 each matrix having 4 elements, is

(a) 3×2^4 (b) 2×4^4 (c) 3×4^4 (d) none of these

Q 49. Let A be a set of n(≥3) distinct elements. The number of triplets (x, y, z) of the elements of A in which at least two coordinates are equal is

(a) ${}^nP_3$ (b) $n^3 - {}^nP_3$ (c) $3n^2 - 2n$ (d) $3n^2(n - 1)$

Q 50. The number of different pairs of word ($\square\square\square\square\ \square\square\square$) that can be made with the letters of the word STATICS is

(a) 828 (b) 1260 (c) 396 (d) none of these

Q 51. Total number of 6-digit numbers in which all the odd digits and only odd digits appear, is

(a) $\frac{5}{6}(6!)$ (b) 6! (c) $\frac{1}{2}(6!)$ (d) none of these

Q 52. The number of divisors of the form 4n + 2 (n ≥ 0) of the integer 240 is

(a) 4 (b) 8 (c) 10 (d) 3

Q 53. In the next World Cup of cricket there will be 12 teams, divided equally in two groups. Teams of each group will play a match against each other. From each group 3 top teams will quality for the next round. In this round each team will play against others once. Four top teams of this round will qualify for the semifinal round, where each team will play against the others once. Two top teams of this round will go to the final round, where they will play the best of three matches. The minimum number of matches in the next World Cup will be

(a) 54 (b) 53 (c) 38 (d) none of these

Q 54. The number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two persons, is

(a) 60(5!) (b) 15(4!) × (5!) (c) 4! × 5! (d) none of these

Q 55. Four couples (husband and wife) decide to form a committee of four members. The number of different committees that can be formed in which no couple finds a place is

(a) 10 (b) 12 (c) 14 (d) 16

Q 56. From 4 gentlemen and 6 ladies a committee of five is to be selected. The number of ways in which the committee can be formed so that gentlemen are in majority is

(a) 66 (b) 156 (c) 60 (d) none of these

Q 57. There are 20 questions in a question paper. If no two students solve the same combination of questions but solve equal number of questions then the maximum number of students who appeared in the examination is

(a) ${}^{20}C_9$ (b) ${}^{20}C_{11}$ (c) ${}^{20}C_{10}$ (d) none of these

Q 58. Nine hundred distinct n-digit positive numbers are to be formed using only the digits 2, 5 and 7. The smallest value of n for which this is possible is

(a) 6 (b) 7 (c) 8 (d) 9

Q 59. The total number of integral solutions for (x, y, z) such that xyz = 24 is

(a) 36 (b) 90 (c) 120 (d) none of these

Q 60. The number of ways in which the letters of the word ARTICLE can be rearranged so that the even places are always occupied by consonants is

(a) 576 (b) ${}^{4}C_3 \times (4!)$ (c) 2(4!) (d) none of these

Q 61. A cabinet of ministers consists of 11 ministers, one minister being the chief minister. A meeting is to be held in a room having a round table and 11 chairs round it, one of them being meant for the chairman. The number of ways in which the ministers can take their chairs, the chief minister occupying the chairman's place, is

(a) $\frac{1}{2}(10!)$ (b) 9! (c) 10! (d) none of these

Q 62. The number of ways in which a couple can sit around a table with 6 guests if the couple take consecutive seats is

(a) 1440 (b) 720 (c) 5040 (d) none of these

Q 63. The number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour, is

(a) $9! \times 10!$ (b) $5(9!)^2$ (c) $(9!)^2$ (d) none of these

Q 64. If $r > p > q$, the number of different selections of $p + q$ things taking r at a time where p things are identical and q other things are identical, is

(a) $p + q - r$ (b) $p + q - r + 1$ (c) $r - p - q + 1$ (d) none of these

Q 65. There are 4 mangoes, 3 apples, 2 oranges and 1 each of 3 other verieties of fruits. The number of ways of selecting at least one fruit of each king is

(a) 10! (b) 9! (c) 4! (d) none of these

Q 66. The number of proper divisors of $2^p . 6^q . 15^r$ is

(a) $(p + q + 1)(q + r + 1)(r + 1)$ (b) $(p + q + 1)(q + r + 1)(r + 1) - 2$

(c) $(p + q)(q + r)r - 2$ (d) none of these

Q 67. The number of proper divisors of 1800 which are also divisible by 10, is

(a) 18 (b) 34 (c) 27 (d) none of these

Q 68. The number of odd proper divisors of $3^p . 6^m . 21^n$ is

(a) $(p + 1)(m + 1)(n + 1) - 2$ (b) $(p + m + n + 1)(n + 1) - 1$

(c) $(p + 1)(m + 1)(n + 1) - 1$ (d) none of these

Q 69. The number of even proper divisors of 1008 is

(a) 23 (b) 24 (c) 22 (d) none of these

Q 70. In a test there were n questions. In the test 2^{n-i} students gave wrong answers to i questions where $i = 1, 2, 3, \ldots, n$. If the total number of wrong answers given is 2047 then n is

(a) 12 (b) 11 (c) 10 (d) none of these

Q 71. The number of ways to give 16 different things to three persons A, B, C so that B gets 1 more than A and C gets 2 more than B, is

(a) $\frac{16!}{4!5!7!}$ (b) 4! 5! 7! (c) $\frac{16!}{3!5!8!}$ (d) none of these

Q 72. The number of ways to distribute 32 different things equally among 4 persons is

(a) $\frac{32!}{(8!)^3}$ (b) $\frac{32!}{(8!)^4}$ (c) $\frac{1}{4}(32!)$ (d) none of these

Q 73. If 3n different things can be equally distributed among 3 persons in k ways then the number of ways to divide the 3n things in 3 equal groups is

(a) $k \times 3!$ (b) $\frac{k}{3!}$ (c) $(3!)^k$ (d) none of these

Q 74. In a packet there are m different books, n different pens and p different pencils. The number of selections of at least one article of each type from the packet is

(a) $2^{m+n+p} - 1$ (b) $(m + 1)(n + 1)(p + 1) - 1$

(c) 2^{m+n+p} (d) none of these

Q 75. The number of 6-digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits is

(a) 480 (b) 540 (c) 1080 (d) none of these

Q 76. The number of words of four letters containing equal number of vowels and consonants, repetition being allowed, is

(a) 105^2 (b) 210×243 (c) 105×243 (d) none of these

Q 77. The number of ways in which 6 different balls can be put in two boxes of different sizes so that no box remains empty is

(a) 62 (b) 64 (c) 36 (d) none of these

Q 78. A shopkeeper selling three varieties of perfumes and he has a large number of bottles of the same size of each variety in his stock. There are 5 places in a row in his showcase. The number of different ways of displaying the three varieties of perfumes in the show case is

(a) 6 (b) 50 (c) 150 (d) none of these

Q 79. The number of arrangements of the letters of the word BHARAT taking 3 at a time is

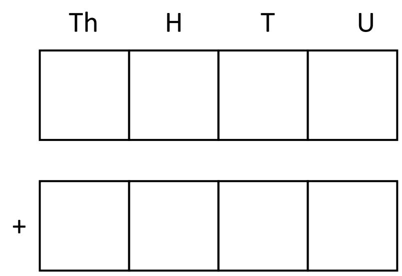
(a) 72 (b) 120 (c) 14 (d) none of these

Q 80. The number of ways to fill each of the four cells of the table with a distinct natural number such that the sum of the number is 10 and the sums of the numbers placed diagonally are equal, is

(a) $2! \times 2!$ (b) $4!$

(c) $2(4!)$ (d) none of these

Q 81. In the figure, two 4-digit numbers are to be formed by filling the places with digits. The number of different ways in which the places can be filled by digits so that the sum of the numbers formed is also a 4-digit number and in no place the addition is with carrying, is



(a) 55^4 (b) 220 (c) 45^4 (d) none of these

Q 82. The number of positive integral solutions of $x + y + z = n$, $n \in N$, $n \geq 3$, is

(a) ${}^{n-1}C_2$ (b) ${}^{n-1}P_2$ (c) $n(n-1)$ (d) none of these

Q 83. The number of non-negative integral solutions of $a + b + c + d = n$, $n \in N$, is

(a) ${}^{n+3}P_2$ (b) $\frac{(n+1)(n+2)(n+3)}{6}$ (c) ${}^{n-1}C_{n-4}$ (d) none of these

Q 84. The number of points (x, y, z) in space, whose each coordinate is a negative integer such that $x + y + z + 12 = 0$, is

(a) 385 (b) 55 (c) 110 (d) none of these

Q 85. If a, b, c are three natural number in AP and $a + b + c = 21$ then the possible number of values of the ordered triplet (a, b, c) is

(a) 15 (b) 14 (c) 13 (d) none of these

Q 86. If a, b, c, d are odd natural number such that $a + b + c + d = 20$ then the number of values of the ordered quadruplet (a, b, c, d) is

(a) 165 (b) 455 (c) 310 (d) none of these

Q 87. If x, y, z are integers and $x \geq 0$, $y \geq 1$, $z \geq 2$, $x + y + z = 15$ then the number of values of the ordered triplet (x, y, z) is

(a) 91 (b) 455 (c) ${}^{17}C_{15}$ (d) none of these

Q 88. If a, b, c are positive integers such that $a + b + c \leq 8$ then the number of possible values of the ordered triplet (a, b, c) is

(a) 84 (b) 56 (c) 83 (d) none of these

Q 89. The number of different ways of distributing 10 marks among 3 questions, each question carrying at least 1 mark, is

(a) 72 (b) 71 (c) 36 (d) none of these

Q 90. The number of ways to give away 20 apples to 3 boys, each boy receiving at least 4 apples, is

(a) ${}^{10}C_8$ (b) 90 (c) ${}^{22}C_{20}$ (d) none of these

Q 91. The position vector of a point P is $\vec{r} + x\vec{i} + y\vec{j} + z\vec{k}$, where $x \in N$, $y \in N$, $z \in N$ and $\vec{a} + \vec{i} + \vec{j} + \vec{k}$. If $\vec{r}.\vec{a} = 10$, the number of possible positions of P is

(a) 36 (b) 72 (c) 66 (d) none of these

Choose the correct options. One or more options may be coorect.

Q 92. If $P = n(n^2 - 1^2)(n^2 - 2^2)(n^2 - 3^2) \dots (n^2 - r^2)$, $n > r$, $n \in N$, then P is divisible by

(a) $(2r + 2)!$ (b) $(2r - 1)!$ (c) $(2r + 1)!$ (d) none of these

Q 93. If ${}^{n+5}P_{n+1} = \frac{11(n-1)}{2} \cdot {}^{n+3}P_n$ then value of n is

(a) 7 (b) 8 (c) 6 (d) 5

Q 94. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in AP then n is

(a) 8 (b) 9 (c) 14 (d) 7

Q 95. The product of r consecutive integers is divisible by

(a) r (b) $\sum_{k=1}^{r-1} k$ (c) r! (d) none of these

Q 96. There are 10 bags $B_1, B_2, B_3, \dots, B_{10}$, which contain 21, 22, ….., 30 different articles respectively. The total number of ways to bring out 10 articles from a bag is

(a) ${}^{31}C_{20} - {}^{21}C_{10}$ (b) ${}^{31}C_{21}$ (c) ${}^{31}C_{20}$ (d) none of these

Q 97. If the number of arrangements of n – 1 things taken from n different things is k times the number of arrangements of n – 1 thing taken from n things in which two things are identical then value of k is

(a) $\frac{1}{2}$ (b) 2 (c) 4 (d) none of these

Q 98. Kanchan has 10 friends among whom two are married to each other. She wishes to invite 5 of them for a party. If the married couple refuse to attend separately then the number of different ways in which she can invite friends is

(a) ${}^8C_5$ (b) $2 \times {}^8C_3$ (c) ${}^{10}C_5 - 2 \times {}^8C_4$ (d) none of these

Q 99. In a plane there are two families of lines $y = x + r$, $y = -x + r$, where $r \in \{0, 1, 2, 3, 4\}$. The number of squares of diagonals of the length 2 formed by the lines is

(a) 9 (b) 16 (c) 25 (d) none of these

Q 100. There are n seats round a table numbered 1, 2, 3, …., n. The number of ways in which $m(\leq n)$ persons can take seats is

(a) ${}^nP_m$ (b) ${}^nC_m \times (m - 1)!$ (c) ${}^{n-1}P_{m-1}$ (d) ${}^nC_m \times m!$

Q 101. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and let $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i}, \vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a} \leq$ 12 then the number of values of $\vec{r}$ is

(a) $^{12}C_9 - 1$ (b) $^{12}C_3$ (c) $^{12}C_9$ (d) none of these

Q 102. The total number of ways in which a beggar can be given at least one rupee from four 25-paisa coins, three 50-paisa coins and 2 one-rupee coins, is

(a) 54 (b) 53 (c) 51 (d) none of these

Q 103. For the equation x + y + z + w = 19, the number of positive integral solutions is equal to

(a) the number of ways in which 15 identical things can be distributed among 4 persons

(b) the number of ways in which 19 identical things can be distributed among 4 persons

(c) coefficient of x^{19} in $(x^0 + x^1 + x^2 + ... + x^{19})^4$ (b) coefficient of x^{19} in $(x + x^2 + x^3 +.....+ x^{19})^4$

Answer

1b	**2a**	**3c**	**4b**	**5a**	**6d**	**7c**	**8a**	**9b**	**10d**
11d	**12a**	**13c**	**14c**	**15b**	**16a**	**17b**	**18c**	**19b**	**20a**
21b	**22c**	**23a**	**24b**	**25b**	**26c**	**27a**	**28c**	**29b**	**30d**
31c	**32b**	**33a**	**34a**	**35c**	**36c**	**37b**	**38a**	**39c**	**40b**
41a	**42c**	**43a**	**44b**	**45d**	**46c**	**47b**	**48c**	**49c**	**50b**
51a	**52a**	**53b**	**54a**	**55d**	**56a**	**57c**	**58b**	**59c**	**60a**
61c	**62a**	**63b**	**64b**	**65c**	**66b**	**67a**	**68b**	**69a**	**70b**
71a	**72b**	**73b**	**74a**	**75c**	**76b**	**77a**	**78c**	**79a**	**80d**
81d	**82a**	**83b**	**84b**	**85c**	**86a**	**87a**	**88b**	**89c**	**90a**
91a	**92bc**	**93ac**	**94cd**	**95abc**	**96a**	**97b**	**98bc**	**99a**	**100ad**
101bc	**102a**	**103ad**							

Determinants and Cramer's Rule

Choose the most appropriate option (a, b, c or d).

Q 1. If $\begin{vmatrix} a+x & a & x \\ a-x & a & x \\ a-x & a & -x \end{vmatrix} = 0$ then x is

(a) 0 (b) a (c) 3 (d) 2a

Q 2. $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix}$ is equal to

(a) p + q + r (b) 0 (c) p – q – r (d) –p + q + r

Q 3. If $a \neq b \neq c$ such that $\begin{vmatrix} a^3-1 & b^3-1 & c^3-1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = 0$ then

(a) ab + bc + ca = 0 (b) a + b + c = 0 (c) abc = 1 (d) a + b + c = 1

Q 4. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$ is equal to

(a) $x^2(x + 3)$ (b) $3x^3$ (c) 0 (d) x^3

Q 5. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$ then

(a) x = 3, y = 1 (b) x = 1, y = 3 (c) x = 0, y = 3 (d) x = 0, y = 0

Q 6. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ for all $p \in R$ if

(a) x, y, z are in AP (b) x, y, z are in GP (c) x, y, z are in HP (d) xy, yz, zx are in AP

Q 7. The determinant $\begin{vmatrix} a & a+d & a+2d \\ a^2 & (a+d)^2 & (a+2d)^2 \\ 2a+3d & 2(a+d) & 2a+d \end{vmatrix} = 0$. Then

(a) d = 0 (b) a + d = 0 (c) d = 0 or a + d = 0 (d) none of these

Q 8. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b, c are the pth, qth and rth terms of a HP, is

(a) ap + bq + cr (b) (a + b + c)(p + q + r) (c) 0 (d) none of these

Q 9. The sum of two nonintegral roots of $\begin{vmatrix} x & 2 & 5 \\ 3 & x & 3 \\ 5 & 4 & x \end{vmatrix} = 0$ is

(a) 5 (b) -5 (c) -18 (d) none of these

Q 10. If x, y, z are integers in AP, lying between 1 and 9, and x51, y41 and z31 are three-digit numbers then the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is

(a) x + y + z (b) x – y + z (c) 0 (d) none of these

Q 11. If $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix}$ then

(a) $\Delta_1 + \Delta_2 = 0$ (b) $\Delta_1 + 2\Delta_2 = 0$ (c) $\Delta_1 = \Delta_2$ (d) none of these

Q 12. Two nonzero distinct numbers a, b are used as elements to make determinants of the third order. The number of determinants whose value is zero for all a, b is

(a) 24 (b) 32 (c) a + b (d) none of these

Q 13. The value of $\begin{vmatrix} a_1x+b_1y & a_2x+b_2y & a_3x+b_3y \\ b_1x+a_1y & b_2x+a_2y & b_3x+a_3y \\ b_1x+a_1 & b_2x+a_2 & b_3x+a_3 \end{vmatrix}$ is equal to

(a) $x^2 + y^2$ (b) 0 (c) $a_1a_2a_3x^2 + b_1b_2b_3y^2$ (d) none of these

Q 14. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$ then the two triangles whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) and (a_1, b_1), (a_2, b_2), (a_3, b_3) are

(a) congruent (b) similar (c) equal in area (d) none of these

Q 15. If α, β are nonreal numbers satisfying $x^3 - 1 = 0$ then the value of

$$\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$$ is equal to

(a) 0 (b) λ^3 (c) $\lambda^3 + 1$ (d) none of these

Q 16. The value of $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix}$ is equal to zero when m is

(a) 6 (b) 4 (c) 5 (d) none of these

Q 17. If x > 0 and $\neq 1$, y > 0 and $\neq 1$, z > 0 and $\neq 1$ then the value of

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$ is

(a) 0 (b) 1 (c) -1 (d) none of these

Q 18. The value of $\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$ is

(a) 0 (b) 30^x (c) 30^{-x} (d) none of these

Q 19. The value of the determinant $\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix}$ is

(a) 0 (b) –(6!) (c) 80 (d) none of these

Q 20. $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ has the value

(a) 0 (b) 1 (c) sinA sinB cos C (d) none of these

Q 21. The value of $\begin{vmatrix} x & x^2-yz & 1 \\ y & y^2-zx & 1 \\ z & z^2-xy & 1 \end{vmatrix}$ is

(a) 1 (b) -1 (c) 0 (d) – xyz

Q 22. If $\sqrt{-1} = i$, and ω is a nonreal cube root of unity then the value of

$$\begin{vmatrix} 1 & \omega^2 & 1+i+\omega^2 \\ -i & -1 & -1-i+\omega \\ 1-i & \omega^2-1 & -1 \end{vmatrix}$$ is equal to

(a) 1 (b) i (c) ω (d) 0

Q 23. If $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x^2-1) \end{vmatrix}$ then f(100) is equal to

(a) 0 (b) 1 (c) 100 (d) -100

Q 24. The value of $\begin{vmatrix} i^m & i^{m+1} & i^{m+2} \\ i^{m+5} & i^{m+4} & i^{m+3} \\ i^{m+6} & i^{m+7} & i^{m+8} \end{vmatrix}$, where $i=\sqrt{-1}$, is

(a) 1 if m is a multiple of 4 (b) 0 for all real m

(c) –i if m is a multiple of 3 (d) none of these

Q 25. If $\Delta_1 = \begin{vmatrix} 7 & x & 2 \\ -5 & x+1 & 3 \\ 4 & x & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} x & 2 & 7 \\ x+1 & 3 & -5 \\ x & 7 & 4 \end{vmatrix}$ then Δ_1 - Δ_2 = 0 for

(a) x = 2 (b) all real x (c) x = 0 (d) none of these

Q 26. If $\Delta_1 = \begin{vmatrix} 10 & 4 & 3 \\ 17 & 7 & 4 \\ 4 & -5 & 7 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 4 & x+5 & 3 \\ 7 & x+12 & 4 \\ -5 & x-1 & 7 \end{vmatrix}$

such that $\Delta_1 + \Delta_2 = 0$ then

(a) x = 5 (b) x has no real value (c) x = 0 (d) none of these

Q 27. Let $\begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix} = p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t$

be an identity in λ, where p, q, r, s, t are independent of λ. Then the value of t is

(a) 4 (b) 0 (c) 1 (d) none of these

Q 28. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5 + bx^4 + cx^3 + dx^2 + \lambda x + \mu$

be an identity in x, where a, b, c, d, λ, μ are independent of x. Then the value of λ is

(a) 3 (b) 2 (c) 4 (d) none of these

Q 29. Using the factor theorem it is found that b + c, c + a and a + b are three factors of the determinant $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$. The other factor in the value of the determinant is

(a) 4 (b) 2 (c) a + b + c (d) none of these

Q 30. If the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of sin x then the constant term in the expansion is

(a) 1 (b) 2 (c) -1 (d) none of these

Q 31. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$ then $\leftarrow \int_0^{\pi/2} \Delta(x)dx$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{2}$

Q 32. If $i = \sqrt{-1}$ and $\sqrt[4]{1} = \alpha, \beta, \gamma, \delta$ then $\begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ is equal to

(a) i (b) –i (c) 1 (d) 0

Q 33. The roots of $\begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & \nu & 1 \end{vmatrix} = 0$ are independent of

(a) λ, μ, ν (b) a, b (c) λ, μ, ν, a, b (d) none of these

Q 34. The value of $\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 4 & 4 & 3 & 0 & 0 \\ 5 & 5 & 5 & 4 & 0 \\ 6 & 6 & 6 & 6 & 5 \end{vmatrix}$ is

(a) 6! (b) 5! (c) $1.2^2 . 3. 4^3. 5^4. 6^4$ (d) none of these

Q 35. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$ = square of a determinant Δ of the third order then Δ is equal to

(a) $\begin{vmatrix} 0 & c & b \\ c & 0 & b \\ b & a & 0 \end{vmatrix}$ (b) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ (c) $\begin{vmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$ (d) none of these

Q 36. The system of equation ax + 4y + z = 0, bx + 3y + z = 0, cx + 2y + z = 0 has nontrivial solutions if a, b, c are in

(a) AP (b) GP (c) HP (d) none of these

Q 37. If the equations a(y + z) = x, b(z + x) = y and c(x + y) = z, where $a \neq -1$, $b \neq -1$, $c \neq -1$, admit of nontrivial solutions then

$(1 + a)^{-1} + (1 + b)^{-1} + (1 + c)^{-1}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) none of these

Q 38. The system of equations

$2x - y + z = 0$

$x - 2y + z = 0$

$\lambda x - y + 2z = 0$

has infinite number of nontrivial solutions for

(a) $\lambda = 1$ (b) $\lambda = 5$ (c) $\lambda = -5$ (d) no real value of λ

Q 39. The equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + mz = n give infinite number of values of the triplet (x, y, z) if

(a) m = 3, $n \in R$ (b) m = 3, $n \neq 10$ (c) m = 3, n = 10 (d) none of these

Q 40. The system of equations 2x + 3y = 8, 7x – 5y + 3 = 0, 4x – 6y + λ = 0 is

(a) 6 (b) 8 (c) -8 (d) -6

Q 41. If the system of equations

ax + by + c = 0

$bx + cy + a = 0$

$cx + ay + b = 0$

has a solution then the system of equations

$(b + c)x + (c + a)y + (a + b)z = 0$

$(c + a)x + (a + b)y + (b + c)z = 0$

$(a + b)x + (b + c)y + (c + a)z = 0$

has

(a) only one solution (b) no solution

(c) infinite number of solutions (d) none of these

Choose the correct options. One or more options may be correct.

Q 42. Let $\{\Delta_1, \Delta_2, \Delta_3, \ldots, \Delta_k\}$ be the set of third order determinants that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, \ldots, a_9$. Then

(a) $k = 9!$ (b) $\sum_{i=1}^{k} \Delta_i = 0$ (c) at least one $\Delta_I = 0$ (d) none of these

Q 43. $\begin{vmatrix} x^2 & (y+z)^2 & yz \\ y^2 & (z+x)^2 & zx \\ z^2 & (x+y)^2 & xy \end{vmatrix}$ is divisible by

(a) $x^2 + y^2 + z^2$ (b) $x - y$ (c) $x - y - z$ (d) $x + y + z$

Q 44. The equation $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = 0$ has

(a) exactly two distinct roots (b) one pair of equal real roots

(c) modulus of each root 1 (d) three pairs of equal roots

Q 45. Let f(n) = $\begin{vmatrix} n & n+1 & n+2 \\ {}^{n}P_{n} & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^{n}C_{n} & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$, where the symbols have their usual meanings. The f(n) is divisible by

(a) $n^2 + n + 1$ (b) $(n + 1)!$ (c) $n!$ (d) none of these

Q 46. Let $x \neq -1$ and let a, b, c be nonzero real numbers. Then the determinant $\begin{vmatrix} a(1+x) & b & c \\ a & b(1+x) & c \\ a & b & c(1+x) \end{vmatrix}$ is divisible by

(a) abcx (b) $(1 + x)^2$ (c) $(1 + x)^3$ (d) $x(1 + x)^2$

Q 47. The arbitrary constant on which the value of the determinant

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \cos(p-d)a & \cos pa & \cos(p-d)a \\ \sin(p-d)a & \sin pa & \sin(p-d)a \end{vmatrix}$$

does not depend is

(a) α (b) p (c) d (d) a

Q 48. Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)dx = -16$, where a, b, c, d are in AP, then the common difference of the AP is

(a) 1 (b) 2 (c) -2 (d) none of these

Q 49. If $A + B + C = \pi$, $e^{i\theta} = \cos\theta + i\sin\theta$ and $z = \begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ then

(a) Re(z) = 4 (b) Im(z) = 0 (c) Re(z) = -4 (d) Im(z) = -1

Q 50. If $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ then x is

(a) 0 (b) a (c) 3a (d) 2a

Q 51. A value of c for which the system of equations

$x + y = 1$

$(c + 2)x + (c + 4)y = 6$

$(c + 2)^2x + (c + 4)^2y = 36$

(a) 1 (b) 2 (c) 4 (d) none of these

Q 52. Eliminating a, b, c from $x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$ we get

(a) $\begin{vmatrix} 1 & -x & x \\ 1 & -y & y \\ 1 & -z & z \end{vmatrix} = 0$ (b) $\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix} = 0$ (c) $\begin{vmatrix} 1 & -x & x \\ y & 1 & -y \\ -z & z & 1 \end{vmatrix} = 0$ (d) none of these

Q 53. The system of equations

$6x + 5y + \lambda z = 0$

$3x - y + 4z = 0$

$x + 2y - 3z = 0$

has

(a) only a trivial solution for $\lambda \in R$

(b) exactly one nontrivial solution for some real λ

(c) infinite number of nontrivial solutions for one value of λ

(d) only one solution for $\lambda \neq -5$

Answers

1a	2b	3c	4a	5d	6b	7c	8c	9b	10c
11a	12b	13b	14c	15b	16c	17a	18a	19b	20a
21c	22d	23a	24b	25b	26a	27b	28a	29a	30c
31d	32d	33b	34b	35a	36a	37a	38c	39c	40b
41c	42ab	43abd	44bcd	45ac	46abd	47b	48bc	49bc	50ac
51bc	52bc	53cd							

Matrices

Choose the most appropriate option (a, b, c or d).

Q 1. If $A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & 3 & 2 \\ 3 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & 4 \\ 1 & 3 & 2 \\ -1 & 1 & 5 \end{bmatrix}$ then A + B is

(a) $\begin{bmatrix} 1 & -2 & 4 \\ 3 & 3 & 2 \\ 2 & 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 8 \\ 3 & 3 & 4 \\ 2 & 1 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -4 & 8 \\ 3 & 6 & 4 \\ 2 & 2 & 10 \end{bmatrix}$ (d) none of these

Q 2. If $A^2 = 8A + kI$ where $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ then k is

(a) 7 (b) -7 (c) 1 (d) -1

Q 3. The matrix $\begin{bmatrix} \lambda & 7 & -2 \\ 4 & 1 & 3 \\ 2 & -1 & 2 \end{bmatrix}$ is a singular matrix if λ is

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) -5 (d) none of these

Q 4. If the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then A^2 is

(a) $\begin{bmatrix} a^2 & b^2 \\ c^2 & d^2 \end{bmatrix}$ (b) $\begin{bmatrix} a^2+bc & ab+bd \\ ac+dc & bc+d^2 \end{bmatrix}$ (c) nonexistent (d) none of these

Q 5. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ such that $A^2 = B$ then α is

(a) 1 (b) -1 (c) 4 (d) none of these

Q 6. If $\begin{bmatrix} 2 & -3 \\ 1 & \lambda \end{bmatrix} \times \begin{bmatrix} 1 & 5 & \mu \\ 0 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & -1 & 13 \end{bmatrix}$ then

(a) $\lambda = 3, \mu = 4$ (b) $\lambda = 4, \mu = -3$

(c) no real values of λ, μ are possible (d) none of these

Q 7. If AB = 0 where $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $|\theta - \phi|$ is equal to

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

Q 8. If $A = \begin{bmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{bmatrix}$ then A^{-1} exists (i.e., A is invertible) if

(a) $\lambda \neq 4$ (b) $\lambda \neq 8$ (c) $\lambda = 4$ (d) none of these

Q 9. The reciprocal matrix of $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} -3 & -4 & 2 \\ -1 & 1 & -1 \\ 1 & 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 4 & -2 \\ 1 & -1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & -1 & 1 \\ -4 & 1 & 2 \\ 2 & -1 & -1 \end{bmatrix}$ (d) none of these

Q 10. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{bmatrix}$ then the value of |adj A| is equal to

(a) 5 (b) 0 (c) 1 (d) none of these

Q 11. If $A = \begin{bmatrix} \cos\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then A^{-1} is equal to

(a) A^T (b) A (c) adj A (d) none of these

Q 12. If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$ the A^2 is equal to

(a) A (b) I (c) A^T (d) none of these

Q 13. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then f(x + y) is equal to

(a) f(x) + f(y) (b) f(x) – f(y) (c) f(x) . f(y) (d) none of these

Q 14. If $A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix}, B = \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix}$ where ω is the complex cube root of 1 then (A + B)C is equal to

(a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Q 15. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ba & b^2 & bc \\ ca & cb & c^2 \end{bmatrix}$ then AB is equal to

(a) 0 (b) I (c) 2I (d) none of these

Q 16. If A be a matrix such that $A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ then A is

(a) $\begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) none of these

Q 17. The rank of the matrix $\begin{bmatrix} -5 & 3 & 2 \\ 3 & 2 & -5 \\ 4 & -1 & -3 \end{bmatrix}$ is

(a) 3 (b) 2 (c) 1 (d) none of these

Q 18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ \lambda & 2 & 4 \\ 2 & -3 & 1 \end{bmatrix}$ is 3 if

(a) $\lambda \neq \frac{18}{11}$ (b) $\lambda = \frac{18}{11}$ (c) $\lambda = -\frac{18}{11}$ (d) none of these

Q 19. The rank of the matrix $\begin{bmatrix} 4 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{bmatrix}$ is

(a) 4 (b) 3 (c) 2 (d) none of these

Q 20. The system of equations

$x + y + z = 2$

$2x - y + 3z = 5$

$x - 2y - z + 1 = 0$

written in matrix form is

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ (d) none of these

Q 21. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ then x is

(a) 2 (b) -2 (c) 14 (d) none of these

Q 22. If $\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ then x.y is equal to

(a) -5 (b) 5 (c) 4 (d) 6

Choose the correct options. One or more options may be correct.

Q 23. $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a

(a) rectangular matrix (b) singular matrix (c) square matrix (d) nonsingular matrix

Q 24. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 0 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 & 6 \\ 4 & 1 & 2 \\ -5 & -1 & 1 \end{bmatrix}$ then

(a) A + B exists (b) AB exists (c) BA exists (d) none of these

Q 25. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then

(a) $A^3 = 9A$ (b) $A^3 = 27A$ (c) $A + A = A^2$ (d) A^{-1} does not exist

Answers

1c	2b	3a	4b	5d	6d	7b	8b	9a	10c
11c	12b	13c	14a	15a	16c	17b	18a	19b	20c
21b	22a	23cd	24c	25acd					